More problems for section 11.5 of *Calculus, Early Transcendentals* by James Stewart, 8e.

The Alternating Series Test tells us that, if $\{b_k\}$ is decreasing and goes to zero, then $\sum_{k=0}^{\infty}(-1)^k b_k$ converges. Consequently, if ℓ is any integer, these alternating series also converge:

$$\sum_{k=\ell}^{\infty}(-1)^k b_k \qquad \sum_{k=\ell}^{\infty}(-1)^{k+1} b_k$$

The AST goes on to say how their partial sums approximate the sum of these series.

Let s be the sum of the series, let c_n be the nth term of the series (either $(-1)^n b_n$ or $(-1)^{n+1} b_n$), and let s_n be the nth partial sum $\sum_{k=\ell}^{n} c_n$. Then

a. $|s - s_n| \leq b_{n+1}$, and

b. $s - s_n$ has the same sign as c_{n+1}.

That is, if $c_{n+1} < 0$, then $s < s_n$, and if $c_{n+1} > 0$, then $s > s_n$.

1. Each of the given series is convergent by the AST.
i. Bound the absolute error that occurs when we approximate the sum s of the series with its 10th partial sum s_{10}
ii. Is s_{10} an overestimate or an underestimate of s?
iii. If we want to approximate s with an absolute error less than 10^{-8} using a partial sum of n terms, how large must n be?

a. $\sum_{k=1}^{\infty} \frac{(-1)^k}{k^{1.5}}$ b. $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2}$ c. $\sum_{k=1}^{\infty} \frac{(-1)^k}{k^3}$

d. $\sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k!}$ e. $\sum_{k=1}^{\infty} \frac{(-1)^{k+2}}{\ln(k+1)}$ f. $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2+1}$

Answers

1a. i. $|s - s_{10}| \leq 11^{-1.5}$, ii. Overestimate, iii. $-1 + 10^{16/3} < n$ 1b. i. $|s - s_{10}| \leq 11^{-2}$, ii. Underestimate, iii. $9999 < n$ 1c. i. $|s - s_{10}| \leq 11^{-3}$, ii. Overestimate, iii. $-1 + 10^{8/3} < n$ 1d. i. $|s - s_{10}| \leq \frac{1}{11!} \approx 2.5 * 10^{-8}$, ii. Underestimate, iii. $11 \leq n$ 1e. i. $|s - s_{10}| \leq \frac{1}{\ln 12} \approx 0.379$, ii. Overestimate, iii. $-2 + e^{10^8} < n$ 1f. i. $|s - s_{10}| \leq \frac{1}{123}$, ii. Underestimate, iii. $-1 + \sqrt{10^8 - 1} < n$