

More problems for section 11.3 of *Calculus, Early Transcendentals* by James Stewart, 8e.

1. Each of the given series is convergent by the integral test. Express your answers to the following as inequalities. Use [2], p. 723.

i. If we approximate the sum s of the series by its 10th partial sum s_{10} , bound the resulting error

$$R_{10} = s - s_{10}$$

above and below.

ii. If we want to approximate s with the n th partial sum s_n , and we need the error

$$R_n = s - s_n$$

to be less than 10^{-6} , how large must n be?

a. $\sum_{k=1}^{\infty} \frac{1}{k^{1.5}}$

b. $\sum_{k=1}^{\infty} \frac{1}{k^2}$

c. $\sum_{k=1}^{\infty} \frac{1}{k^3}$

d. $\sum_{k=1}^{\infty} \frac{1}{k^5}$

e. $\sum_{k=1}^{\infty} \frac{1}{k^7}$

f. $\sum_{k=1}^{\infty} \frac{1}{k^2+1}$

Answers

1a. i. $0.603 < R_{10} < 0.632$, ii. $4 \cdot 10^{12} \leq n$ 1b. i. $0.909 < R_{10} < 0.100$, ii. $10^6 \leq n$ 1c. i. $0.004 < R_{10} < 0.005$, ii. $708 \leq n$ 1d. i. $1.7 \cdot 10^{-5} < R_{10} < 2.5 \cdot 10^{-5}$, ii. $23 \leq n$ 1e. i. $9.4 \cdot 10^{-8} < R_{10} < 1.7 \cdot 10^{-7}$, ii. $8 \leq n$ 1f. i. $\frac{\pi}{2} - \arctan(11) < R_{10} < \frac{\pi}{2} - \arctan(10)$, ii. $\tan(\pi/2 - 10^{-6}) \leq n$