0.1: The Binomial Theorem and Pascal's Triangle.

The formulas

$$(x+y)^2 = x^2 + 2xy + y^2, \text{ and}$$
$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3.$$

are special instances of the **Binomial Theorem**, which says that the coefficients in the expansion

$$(x+y)^n = x^n + ?x^{n-1}y + ?x^{n-2}y^2 + \cdots + ?x^2y^{n-2} + ?xy^{n-1} + y^n$$

are found in **Pascal's Triangle**:

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

Details can be found in http://kunklet.people.cofc.edu/MATH111/pascal.pdf

0.1.re1.

$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$
$$(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$

0.1.re2. Generate the next three rows of Pascal's Triangle.

0.1.re3. Expand the following.

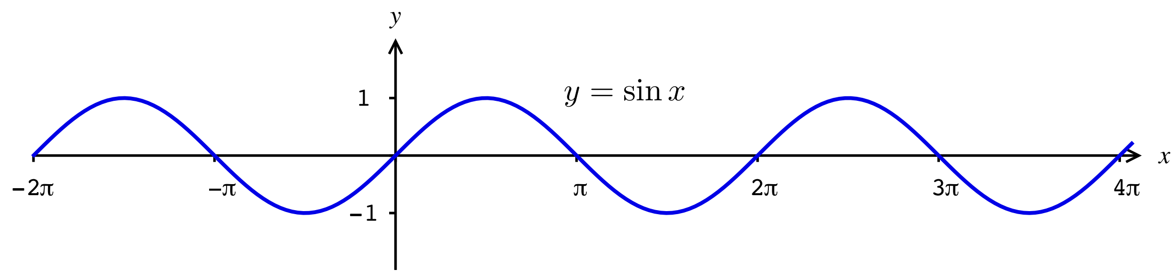
a. $(x+3)^4$ b. $(u+v)^5$ c. $(u-v)^6$
d. $(x^3+y)^4$ e. $(x-x^{-1})^5$ f. $(\xi-2)^6$

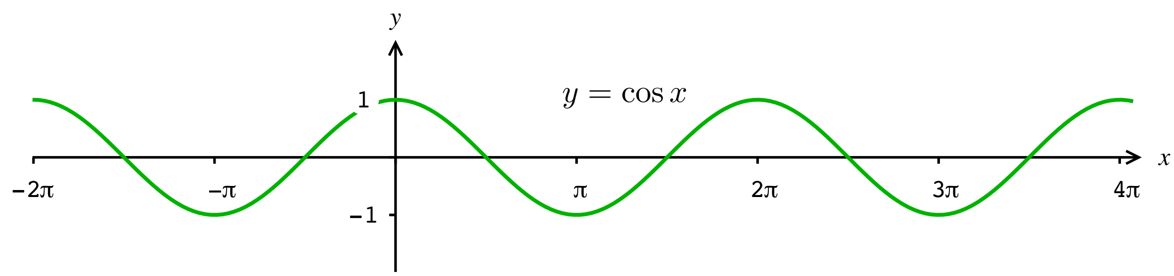
Answers

0.1.re2. 5th row: 1 5 10 10 5 1. 6th row: 1 6 15 20 15 6 1. 7th row: 1 7 21 35 35 21 7 1.
0.1.re3a. $x^4+12x^3+54x^2+108x+81$ 0.1.re3b. $u^5+5u^4v+10u^3v^2+10u^2v^3+5uv^4+v^5$
0.1.re3c. $u^6-6u^5v+15u^4v^2-20u^3v^3+15u^2v^4-6uv^5+v^6$ 0.1.re3d. $x^{12}+4x^9y+6x^6y^2+4x^3y^3+y^4$
0.1.re3e. $x^5-5x^3+10x-10x^{-1}+5x^{-3}+x^{-5}$ 0.1.re3f. $\xi^6-12\xi^5+60\xi^4-160\xi^3+240\xi^2-192\xi+64$

Ap.D: Trigonometry

For a more complete review of trignometry, see Appendix D of our text.
The two basic functions in trigonometry are the sine and cosine, graphed here:



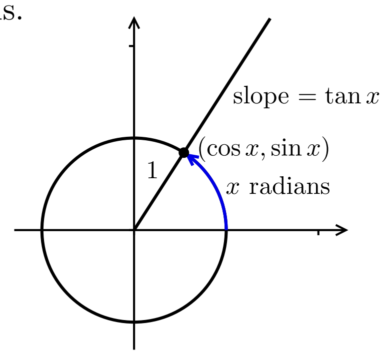


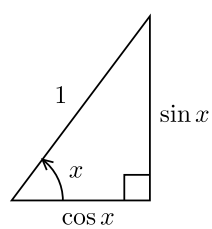
The other four trig functions are defined using sine and cosine:

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

$$\sec x = \frac{1}{\cos x} \qquad \csc x = \frac{1}{\sin x}$$

$\sin x$ and $\cos x$ are defined for all real numbers x, but $\tan x$ and $\sec x$ are undefined whenever $\cos x = 0$, and $\cot x$ and $\csc x$ are undefined whenever $\sin x = 0$.
By definition, $\cos x$ and $\sin x$ are the coordinates of the point on the **unit circle** (i.e., the circle of radius one centered at the origin) x radians counterclockwise from the positive horizontal axis.

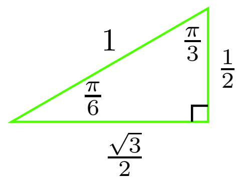


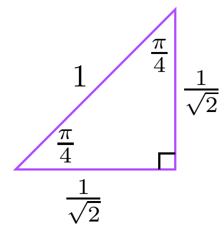


Consequently, the ray through the origin x radians from the positive horizontal axis has slope $\tan x$, and, when x is an acute angle, $\cos x$ and $\sin x$ are the legs of this right triangle with hypotenuse 1 and interior angle x

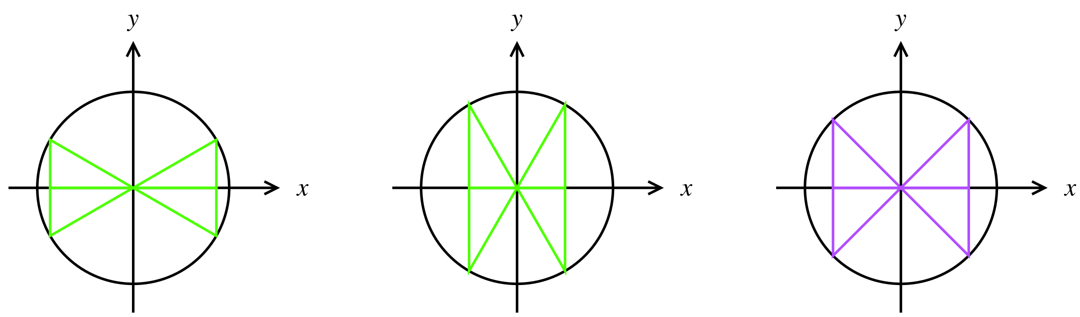
Known values of sine and cosine

We already know the values of sine and cosine at the four points where the unit circle intersects the x and y axes. By placing these two triangles:



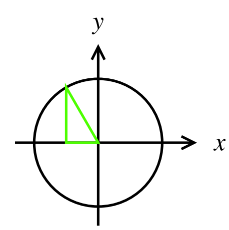


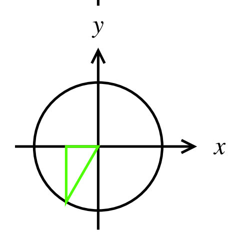
around the unit circle like this:



we find the sines and cosines at 12 more points on the unit circle (and the infinitely many angles that reach those points).

Ap.D.re1. Find all angles whose cosine is $-\frac{1}{2}$.





We recognize $1/2$ as the short side of the 30-60-90 triangle, so for the cosine to be $-1/2$, the angle must be one of the two pictured at right. Find one angle that matches each drawing, for instance,

$$x = \pi - \pi/3 = 2\pi/3 \quad \text{and} \quad x = \pi + \pi/3 = 4\pi/3.$$

Then add all multiples of 2π to describe *all* angles that fit the drawings:

$$x = 2\pi/3 + 2\pi n \quad \text{and} \quad x = 4\pi/3 + 2\pi n \quad (\text{where } n \text{ is any integer}).$$

Ap.D.re2. Find all solutions x to the given equation.

a. $\sin x = -1/\sqrt{2}$ b. $\cos x = 0$ c. $\tan x = -1$

d. $\sec x = -2$ e. $\csc x = 2/\sqrt{3}$ f. $\cot x = \sqrt{3}$

Identities

You must know the derivatives of the six trig functions and these trig identities:

PYTHAGOREAN IDENTITIES
$\sin^2 x + \cos^2 x = 1$
$\tan^2 x + 1 = \sec^2 x$
$1 + \cot^2 x = \csc^2 x$
EVEN & ODD IDENTITIES
$\sin(-x) = -\sin x$
$\cos(-x) = \cos x$
SUM & DIFFERENCE IDENTITIES
$\sin(x+y) = \sin x \cos y + \cos x \sin y$
$\cos(x+y) = \cos x \cos y - \sin x \sin y$
$\sin(x-y) = \sin x \cos y - \cos x \sin y$
$\cos(x-y) = \cos x \cos y + \sin x \sin y$
DOUBLE ANGLE FORMULAS
$\sin(2x) = 2 \sin x \cos x$
$\cos(2x) = \cos^2 x - \sin^2 x$
$= 2\cos^2 x - 1$
$= 1 - 2\sin^2 x$
HALF ANGLE FORMULAS
$\cos^2 x = \frac{1}{2}\big(1 + \cos(2x)\big)$
$\sin^2 x = \frac{1}{2}\big(1 - \cos(2x)\big)$

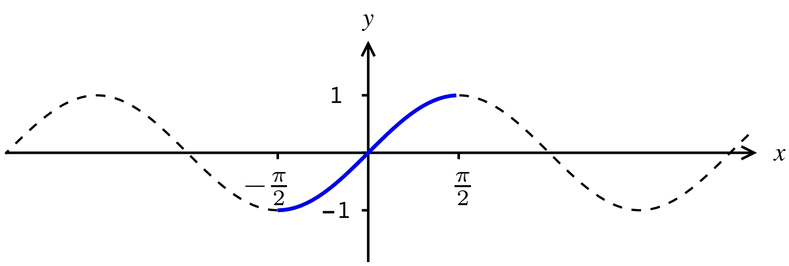
Ap.D.re3. The first pythagorean identity follows from the definition of sine and cosine. Derive the second and third pythagorean identities from the first by dividing by either $\sin^2 x$ or $\cos^2 x$.

Ap.D.re4. The formula for $\cos(x + y)$ is obtained by differentiating both sides of the formula for $\sin(x + y)$ with respect to x assuming y is a constant. Derive the other sum/difference/double/half-angle formulas from these two by differentiating with respect to x, replacing y with $-y$ and using the even/odd identities, or replacing y with x.

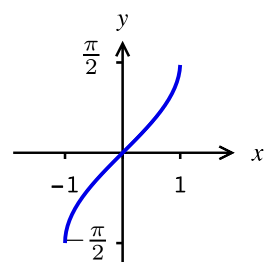
The inverse trig functions

To define inverses of the trig functions, we restrict each to a domain on which it takes each value in its range exactly once. The three we'll see most often in calculus are the inverses of the sine, tangent, and secant.

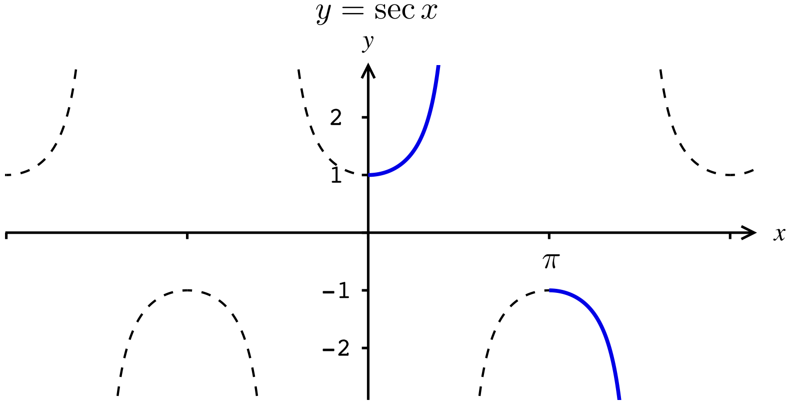
$y = \sin x$



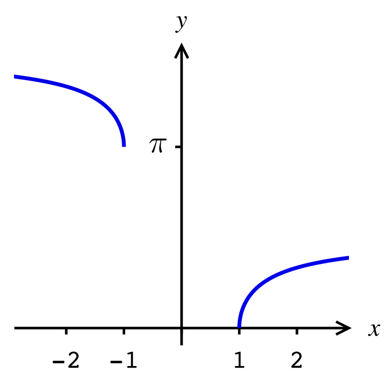
$y = \sin^{-1} x$



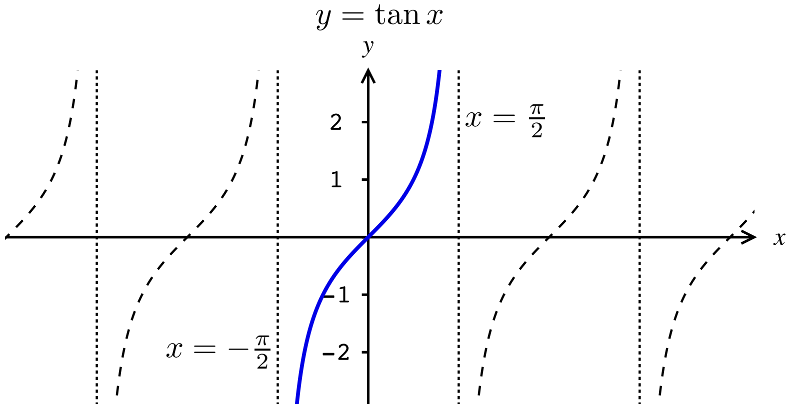
$y = \sec x$



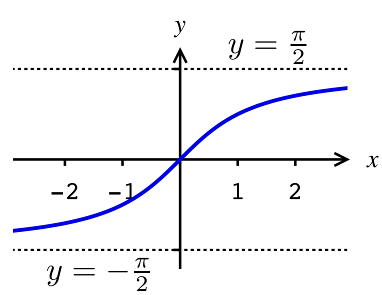
$y = \sec^{-1} x$



$y = \tan x$



$y = \tan^{-1} x$



Traditionally, each of the inverse trig functions has two names. The inverse function of $\sin x$ is called $\sin^{-1} x$ or $\arcsin x$, the inverse function of $\sec x$ is called $\sec^{-1} x$ or $\operatorname{arcsec} x$, etc..

As indicated in the graphs above,

	Domain	Range
$\sin^{-1}$	$[-1,1]$	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$
$\sec^{-1}$	$(-\infty,-1]\cup[1,\infty)$	$\left[0,\frac{\pi}{2}\right)\cup\left[\pi,\frac{3\pi}{2}\right)$
$\tan^{-1}$	$(-\infty,\infty)$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

although there are few times in this course when you'll need to know these details.

Tip: if $x>0$, and "arctrig" is any of the inverse trig functions, then $\operatorname{arctrig} x\in[0,\frac{\pi}{2}]$.

It is helpful to remember the definitions of the inverse trig functions in words:

If $-1\le x\le 1$, then $\sin^{-1}x$ is the angle in $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ whose sine is x. That is,

$$\sin(\sin^{-1}x)=x \quad \text{for any } x\in[-1,1].$$

If $|x|\ge 1$, then $\sec^{-1}x$ is the angle in $\left[0,\frac{\pi}{2}\right)\cup\left[\pi,\frac{3\pi}{2}\right)$ whose secant is x. That is,

$$\sec(\sec^{-1}x)=x \quad \text{for any } x\in(-\infty,-1]\cup[1,\infty).$$

If $-\infty<x<\infty$, then $\tan^{-1}x$ is the angle in $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ whose tangent is x. That is,

$$\tan(\tan^{-1}x)=x \quad \text{for any real number } x.$$

Ap.D.re5. Evaluate: $\sin^{-1}\left(-\frac{1}{2}\right)$

Answers

Ap.D.re2a. $x=-\pi/4+2\pi n$ or $x=-3\pi/4+2\pi n$ Ap.D.re2b. $x=\pi/2+2\pi n$ or $x=-\pi/2+2\pi n$
Ap.D.re2c. $x=3\pi/4+2\pi n$ or $x=11\pi/4+2\pi n$ Ap.D.re2d. (same as in example Ap.D.re1)
Ap.D.re2e. $x=\pi/3+2\pi n$ or $x=2\pi/3+2\pi n$ Ap.D.re2f. $x=\pi/3+2\pi n$ or $x=4\pi/3+2\pi n$ Ap.D.re5. $-\pi/6$

D: Differentiation

The **derivative** of the function $f(x)$, denoted $f'(x)$, is defined to be

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{t\to x} \frac{f(t) - f(x)}{t - x}.$$

The **linearization** of a function $f(x)$ at $x = a$ is the linear function having the same value and derivative value as f at a and is given by

$$L(x) = f(a) + f'(a)(x - a).$$

Differentiation rules allow us to differentiate many functions without resorting to the definition. These rules come in two types.

I. A catalog of elementary functions and their derivatives.

$f(x)$	derivative of f
$f(x)$	$f'(x)$
x^n	nx^{n-1}
$\ln\|x\|$	x^{-1}
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\csc^2 x$
$\sec x$	$\sec x \tan x$
$\csc x$	$-\csc x \cot x$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\sec^{-1} x$	$\frac{1}{x\sqrt{x^2-1}}$

II. Combination laws.

Linearity Rules: If $f(x)$ and $g(x)$ are differentiable and c is a constant, then

$$\big(cf(x)\big)' = cf'(x)$$
$$\big(f(x) + g(x)\big)' = f'(x) + g'(x)$$

Product Rule: If $f(x)$ and $g(x)$ are differentiable, then so is their product, and

$$\big(f(x)g(x)\big)' = f'(x)g(x) + f(x)g'(x).$$

Quotient Rule: If $f(x)$ and $g(x)$ are differentiable, then so is their quotient, and

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2} \qquad \text{(provided } g(x) \neq 0\text{).}$$

Chain Rule: If f and g are differentiable functions, then their composition is differentiable (where it exists), and

$$\Big[f\big(g(x)\big)\Big]' = f'\big(g(x)\big)g'(x)$$

When more than one combination law applies, remember this advice:

The last operation determines the first differentiation rule we must use.

D.re1. Find the indicated derivatives of the given function.

a. $f'(x)$ & $f''(x)$, where $f(x) = 7x^3 - \frac{3}{\sqrt[3]{x^2}} + 2x + 5^2$

b. $g^{(3)}(x)$ & $g^{(4)}(x)$, where $g(x) = 7x^3 + 4x^2 + 2x + 25$

D.re2. Find the quadratic function $k(x)$ satisfying

$$k(2) = 1 \qquad k'(2) = -1 \qquad k''(2) = 6.$$

D.re3. Find the derivative of the given function.

a. $x^2 e^x$ b. $x^2 e^x(2x-3)$ c. $\dfrac{3x^3+2}{x^2}$

d. $e^x\sqrt{x}\sin x$ e. $\dfrac{e^x \sin x}{e^x \tan x}$

D.re4. Find the equation of the line tangent to $y = \dfrac{xe^x}{3x^2-1}$ at $x = 1$.

D.re5. Find the derivative of the given function.

a. $\sin x^2$ b. $\sin^2 x$ c. $\sqrt{x^2-5x+1}$
d. $\ln 2$ e. $\ln(2x)$ f. $\ln(e^{-x}5x^2\sin x)$
g. $\ln|\sec x|$ h. $\log_3 x$ i. $e^{\frac{1}{2}\ln x}$
j. $x^{x/2}$ k. $e^{x\sin x}$ l. 2^x
m. $e^{\tan^2 x}$ n. $x^2\cos^2 x^3$ o. $(\sin^2 x)e^{\cos x}$
p. $\dfrac{e^x}{\sec\sqrt{x}}$ q. $\tan^{-1}(\sin^{-1} x)$ r. $(\tan^{-1} x)(\sin^{-1} x)$
s. $e^x \arccos x^3$ t. $\sec(\sec^{-1} x)$ u. $\cot(\arctan e^x)$

D.re6. Find the linearization of the function at the given point.

a. e^x, $a = 0$ b. e^x, $a = 1$
c. $\sin x$, $a = 0$ d. $\sin x$, $a = \pi/4$
e. $\sqrt{x}$, $a = 9$ f. $e^x(\sin x - \cos x)$

For more differentiation practice, see the review problems from Chapter 3 listed on our syllabus under **Assigned Problems**.

Answers

D.re1a. $f'(x) = 21x^2 + 2x^{-5/3} + 2$. $f''(x) = 42x - \frac{10}{3}x^{-8/3}$. D.re1b. $g'''(x) = 42$. $g^{(4)}(x) = 0$. D.re2. $3x^2 - 13x + 15$ D.re3a. $(x^2+2x)e^x$ D.re3b. $(2x^3+3x^2-3x)e^x$ D.re3c. $3 - 4x^{-3}$
D.re3d. $e^x\sqrt{x}\sin x + \frac{1}{2}e^x\frac{1}{\sqrt{x}}\sin x + e^x\sqrt{x}\cos x$ D.re3e. $-\sin x$ D.re4. $y - \frac{e}{2} = -\frac{e}{2}(x-1)$ D.re5a. $2x\cos(x^2)$
D.re5b. $2\sin x\cos x$ D.re5c. $\frac{1}{2}(2x-5)(x^2-5x+1)^{-1/2}$ D.re5d. 0 D.re5e. x^{-1} D.re5f. $-1 + \frac{2}{x} + \cot x$
D.re5g. $\tan x$ D.re5h. hint: $\log_3 x = (\ln x)/(\ln 3)$. Derivative $= \frac{1}{x\ln 3}$. D.re5i. hint: simplify before differentiating. Derivative is $\frac{1}{2}x^{-1/2}$. D.re5j. hint: $a^b = e^{\ln a^b} = e^{b\ln a}$. Derivative $= \frac{1}{2}e^{\frac{1}{2}x\ln x}(1+\ln x)$.
D.re5k. $(x\cos x + \sin x)e^{x\sin x}$ D.re5l. hint: $2^x = e^{\ln(2^x)} = e^{x\ln 2}$. Derivative $= e^{x\ln 2}\ln 2$.
D.re5m. $e^{\tan^2 x}2\tan x\sec^2 x$ D.re5n. $2x\cos^3(x^3) - 6x^4\cos(x^3)\sin(x^3)$ D.re5o. $(2\sin x\cos x - \sin^3 x)e^{\cos x}$
D.re5p. hint: $\frac{e^x}{\sec\sqrt{x}} = e^x\cos\sqrt{x}$. Derivative $= e^x(\cos(\sqrt{x}) - \frac{1}{2}x^{-1/2}\sin(\sqrt{x}))$. D.re5q. $\frac{1}{(1+(\sin^{-1}x)^2)\sqrt{1-x^2}}$
D.re5r. $\frac{\sin^{-1}x}{1+x^2} + \frac{\tan^{-1}x}{\sqrt{1-x^2}}$ D.re5s. $e^x\left(\cos^{-1}x^3 - \frac{3x^2}{\sqrt{1-x^6}}\right)$ D.re5t. hint: simplify before differentiating. Derivative $= 1$. D.re5u. hint: simplify before differentiating. Derivative is $-e^{-x}$. D.re6a. $1 + x$
D.re6b. xe D.re6c. x D.re6d. $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}(x - \frac{\pi}{4})$ D.re6e. $3 + \frac{1}{6}(x-9)$ D.re6f. $e^{\pi/2}\Big(1 + 2(x-\pi/2)\Big)$

I: Integration

The **definite integral** from a to b of $f(x)\,dx$, written $\int_a^b f(x)\,dx$, is the limit of the Riemann sums of f on the interval $[a,b]$ as the length of the subintervals goes to zero.

The **indefinite integral** of $f(x)\,dx$, written $\int f(x)\,dx$, is the collection of all **antiderivatives** of $f(x)$, that is, those functions $F(x)$ whose derivative $F'(x)$ equals $f(x)$.

$$\begin{aligned}
\int_a^b f(x)\,dx &= \text{a number.} \\
&= \text{the net signed area trapped between the graph of } f(x) \\
& \text{ and the } x\text{-axis from } x=a \text{ to } x=b. \\
&= \text{the net distance traveled from time } x=a \text{ to time } x=b \\
& \text{ by an object whose velocity at time } x \text{ is } f(x). \\
\int f(x)\,dx &= \text{a collection of functions.} \\
&= \text{the set of all antiderivatives of } f(x).
\end{aligned}$$

If $F(x)$ is an antiderivative of $f(x)$ on an interval, then, by the Mean Value Theorem, every antiderivative of f on that interval equals $F(x)+C$ for some constant C:

$$\int f(x)\,dx = F(x)+C, \text{ where } F'(x)=f(x)$$

According to the FTC below, evaluating a definite integral consists primarily of finding an antiderivative. We refer to both of those operations as **integration**.

Fundamental Theorem of Calculus: If $F(x)$ is any antiderivative of $f(x)$ and if f is continuous on $[a,b]$, then

$$\int_a^b f(t)\,dt = F(b)-F(a) \overset{\text{def}}{=} F(x)\Big|_a^b$$

I.re1. Integrate.

a. $\displaystyle\int (x^{0.7}-2e^x)\,dx$ b. $\displaystyle\int_1^2 (1-6t^2+10t^4)\,dt$ c. $\displaystyle\int \frac{r^3+1}{2r}\,dr$

d. $\displaystyle\int_{-\pi/6}^{\pi/4} \sec x\tan x\,dx$ e. $\displaystyle\int_{-3}^{3} x^{11}\,dx$ f. $\displaystyle\int (u+3)(u-4)\,du$

A table of antiderivatives is practically the same as a table of derivatives read backwards:

$f(x)$	derivative of f
$f(x)$	$f'(x)$
x^n	nx^{n-1}
$\ln\lvert x\rvert$	x^{-1}
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\csc^2 x$
$\sec x$	$\sec x \tan x$
$\csc x$	$-\csc x \cot x$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\sec^{-1} x$	$\frac{1}{x\sqrt{x^2-1}}$

$f(x)$	antiderivative of f
$f(x)$	$\int f(x)\,dx$
x^n	$\begin{cases} \frac{1}{n+1}x^{n+1}+C & \text{if } n \neq -1 \\ \ln\lvert x\rvert + C & \text{if } n = -1 \end{cases}$
e^x	$e^x + C$
$\cos x$	$\sin x + C$
$\sin x$	$-\cos x + C$
$\sec^2 x$	$\tan x + C$
$\csc^2 x$	$-\cot x + C$
$\sec x \tan x$	$\sec x + C$
$\csc x \cot x$	$-\csc x + C$
$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1} x + C$
$\frac{1}{1+x^2}$	$\tan^{-1} + C$
$\frac{1}{x\sqrt{x^2-1}}$	$\sec^{-1} x + C$

There are fewer combination rules for antiderivatives than for derivatives. If f and g are functions and c is a constant, then

$$\boxed{\begin{aligned} \int \big(f(x)+g(x)\big)\,dx &= \int f(x)\,dx + \int g(x)\,dx \\ \int cf(x)\,dx &= c\int f(x)\,dx \end{aligned}}$$

Generally, it's difficult to integrate a product or quotient, because

$$\left(\int f(x)\,dx\right)\left(\int g(x)\,dx\right) \neq \int f(x)g(x)\,dx, \quad \text{and}$$
$$\left(\int f(x)\,dx\right) \div \left(\int g(x)\,dx\right) \neq \int \big(f(x)/g(x)\big)\,dx.$$

For example, x^2 is an antiderivative for $2x$, and e^x is an antiderivative for itself, but x^2e^x is **not** an antiderivative for $2xe^x$, and $\dfrac{e^x}{x^2}$ is **not** an antiderivative for $\dfrac{e^x}{2x}$.

I.re2. Find $f(x)$ if

a. $f'(x) = \dfrac{1}{x}$ and $f(-2) = 1$

b. $f''(x) = \sin x - x^2$ and $f'(0) = 2$ and $f(0) = -1$

I.re3. Integrate.

a. $\displaystyle\int (x^2 - 3)\sqrt{x}\,dx$ b. $\displaystyle\int_2^1 \frac{x^2-3}{x^{-2}}\,dx$ c. $\displaystyle\int_0^1 e^p\,dp$

d. $\displaystyle\int_{-6}^{-2} \frac{ds}{s}$ e. $\displaystyle\int_{-1}^{4} e\,dw$ f. $\displaystyle\int (2\csc x\cot x - 3\cos x)\,dx$

g. $\displaystyle\int (x^2-1)^3\,dx$ h. $\displaystyle\int_0^{\pi/4} \frac{1}{\cos^2 t}\,dt$ i. $\displaystyle\int \frac{r^3 - 7r^2 + 1}{2r}\,dr$

j. $\displaystyle\int \frac{x^3+8}{x+2}\,dx$ k. $\displaystyle\int_{-2}^{2} |x|\,dx$ l. $\displaystyle\int_0^{1/2} \frac{du}{\sqrt{1-u^2}}$

m. $\displaystyle\int \left(\frac{2}{x} + \frac{3}{1+x^2}\right)dx$ n. $\displaystyle\int \csc x(\csc x - \cot x)\,dx$

The **Substitution Rule** uses the chain rule to help us to simplify an integral. It says that an integral of the form $\int f\big(u(x)\big)u'(x)\,dx$ (for some function u of x) can be rewritten

$$\int f(u)\frac{du}{dx}\,dx = \int f(u)\,du$$

and thenceforth be treated as if u were the variable of integration. For this reason, substitution is also called a **change of variable**.

I.re4. Integrate.

a. $\displaystyle\int x^3\sin(x^4)\,dx$ b. $\displaystyle\int (2x+1)\sqrt{x^2+x}\,dx$ c. $\displaystyle\int \frac{\sqrt{\ln x}}{x}\,dx$

d. $\displaystyle\int 2^{x+3}\,dx$ e. $\displaystyle\int \cos x\cot(\sin x)\sec^2(\sin x)\,dx$ f. $\displaystyle\int \frac{1}{1+\sqrt{x}}\,dx$

g. $\displaystyle\frac{2e^{\sqrt{x}}}{\sqrt{x}}\,dx$ h. $\displaystyle\int \frac{e^x}{e^{2x}+1}\,dx$ i. $\displaystyle\int (2t+1)^{19}\,dt$

j. $\displaystyle\int x\sqrt{x+2}\,dx$

For more integration practice, see the review problems from Chapter 5 listed on our syllabus under **Assigned Problems**.

Answers

I.re1a. $\frac{1}{1.7}x^{1.7} - 2e^x + C$ I.re1b. 49 I.re1c. $\frac{1}{6}r^3 + \frac{1}{2}\ln|r| + C$ I.re1d. $2+\sqrt{2}$. I.re1e. 0 I.re1f. $\frac{1}{3}u^3 - \frac{1}{2}u^2 - 12u + C$ I.re2a. $f(x) = \ln|x| + 1 - \ln 2$ I.re2b. $f(x) = -\sin x - \frac{1}{12}x^4 + 3x - 1$ I.re3a. $\frac{2}{7}x^{3.5} - 2x^{1.5} + C$ I.re3b. 4/5 I.re3c. $e-1$ I.re3d. $-\ln 3$ I.re3e. $5e$ I.re3f. $-2\csc x - 3\sin x + C$ I.re3g. $\frac{1}{7}x^7 - \frac{3}{5}x^5 + x^3 - x + C$

I.re3h. 1 I.re3i. $\frac{1}{6}r^3 \frac{7}{4}r^2 - \frac{1}{2}\ln|r| + C$ I.re3j. hint: use long division to rewrite integrand. $\frac{1}{3}x^3 - x^2 + 4x + C$. I.re3k. 4 I.re3l. $\pi/6$ I.re3m. $2\ln|x| + 3\tan^{-1} x + C$ I.re3n. $-\cot x + \csc x + C$ I.re4a. $-\frac{1}{4}\cos(x^4) + C$ I.re4b. $\frac{2}{3}(x^2+x)^{3/2} + C$ I.re4c. $\frac{2}{3}(\ln x)^{3/2} + C$ I.re4d. $\frac{1}{\ln 2}2^{x+3} + C$ I.re4e. $\ln|\tan(\sin x)| + C$ I.re4f. $2\sqrt{x} - 2\ln(1+\sqrt{x}) + C$ I.re4g. $4e^{\sqrt{x}} + C$ I.re4h. $\tan^{-1} e^x + C$. I.re4i. $\frac{1}{40}(2t+1)^{20} + C$ I.re4j. $\frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C$