

1. Use Row Reduction to find the RREF of $G = \begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 2 & -2 & -12 \\ 2 & 2 & 5 & 22 \\ -1 & 4 & -2 & -1 \end{bmatrix}$.

Solution:

1. Begin forward phase:

row operation	result
initial matrix	$\begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 2 & -2 & -12 \\ 2 & 2 & 5 & 22 \\ -1 & 4 & -2 & -19 \end{bmatrix}$
$\mathbf{r}_3 \leftarrow \mathbf{r}_3 - 2\mathbf{r}_1$	$\begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 2 & -2 & -12 \\ 0 & -2 & 3 & 16 \\ -1 & 4 & -2 & -19 \end{bmatrix}$
$\mathbf{r}_4 \leftarrow \mathbf{r}_4 + \mathbf{r}_1$	$\begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 2 & -2 & -12 \\ 0 & -2 & 3 & 16 \\ 0 & 6 & -1 & -16 \end{bmatrix}$
$\mathbf{r}_3 \leftarrow \mathbf{r}_3 + \mathbf{r}_2$	$\begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 2 & -2 & -12 \\ 0 & 0 & 1 & 4 \\ 0 & 6 & -1 & -16 \end{bmatrix}$
$\mathbf{r}_4 \leftarrow \mathbf{r}_4 - 3\mathbf{r}_2$	$\begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 2 & -2 & -12 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 5 & 20 \end{bmatrix}$
$\mathbf{r}_4 \leftarrow \mathbf{r}_4 - 5\mathbf{r}_3$	$\begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 2 & -2 & -12 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

End forward phase. Matrix is in row echelon form. Backward phase next.

Begin backward phase

row operation	result
$\mathbf{r}_2 \leftarrow \mathbf{r}_2 + 2\mathbf{r}_3$	$\begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 2 & 0 & -4 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
$\mathbf{r}_2 \leftarrow \frac{1}{2}\mathbf{r}_2$	$\begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
$\mathbf{r}_1 \leftarrow \mathbf{r}_1 - \mathbf{r}_3$	$\begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
$\mathbf{r}_1 \leftarrow \mathbf{r}_1 - 2\mathbf{r}_2$	$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

End backward phase. Matrix is in reduced row echelon form.