MATH 203–03 (Kunkle), Quiz 9 Name: _____
10 pts, 10 minutes Nov 7, 2024

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1 (4 pts). What are the eigenvalues of the matrix

$$B = \begin{bmatrix} -3 & 1 & -2 \\ -2 & 0 & -1 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 9 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \\ 1 & -3 & -1 \\ 0 & -3 & -2 \end{bmatrix}?$$

Give a nonzero eigenvector for each eigenvalue.
Hint:

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & -3 & -1 \\ 0 & -3 & -2 \end{bmatrix} \begin{bmatrix} -3 & 1 & -2 \\ -2 & 0 & -1 \\ 3 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2 (6 pts). Find the characteristic polynomial and eigenvalues of

$$M = \begin{bmatrix} -2 & 3 \\ -5 & 6 \end{bmatrix}$$

Solution:

1.(Source: 5.3.5-6) B has been diagonalized, so its eigenvalues are 9, -6, 5. The corresponding columns of the matrix on the left are eigenvectors:

$$\lambda = 9;\ \mathbf{x} = \begin{bmatrix} -3 \\ -2 \\ 3 \end{bmatrix}. \qquad \lambda = -6;\ \mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad \lambda = 5;\ \mathbf{x} = \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}$$

(done)

To illustrate, the Hint implies that $\begin{bmatrix} 0 & 1 & 1 \\ 1 & -3 & -1 \\ 0 & -3 & -2 \end{bmatrix} \begin{bmatrix} -3 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, and therefore

$$B \begin{bmatrix} -3 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} -3 & 1 & -2 \\ -2 & 0 & -1 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 9 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \\ 1 & -3 & -1 \\ 0 & -3 & -2 \end{bmatrix} \begin{bmatrix} -3 \\ -2 \\ 3 \end{bmatrix}$$
$$= \begin{bmatrix} -3 & 1 & -2 \\ -2 & 0 & -1 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 9 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -3 & 1 & -2 \\ -2 & 0 & -1 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 9 \\ 0 \\ 0 \end{bmatrix} = 9 \begin{bmatrix} -3 \\ -2 \\ 3 \end{bmatrix}.$$

2.(Source: 5.2.1-8) The characteristic polynomial is

$$|M - \lambda I| = \begin{vmatrix} -2-\lambda & 3 \\ -5 & 6-\lambda \end{vmatrix} = (-2-\lambda)(6-\lambda) - (-5)3$$
$$= -12 - 4\lambda + \lambda^2 + 15 = \lambda^2 - 4\lambda + 3 = (\lambda - 3)(\lambda - 1)$$

The eigenvalues are the zeros of the characteristic polynomial, that is, 1 and 3. **(done)**