MATH 203–03 (Kunkle), Quiz 8
20 pts, 10 minutes

Name: ____________________
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1 (10 pts). 2 is an eigenvalue of the matrix

$$E = \begin{bmatrix} 7 & -4 & 4 \\ 9 & -4 & 6 \\ -3 & 3 & -1 \end{bmatrix}.$$

Find a basis for the corresponding eigenspace.

Solution:

1 (10 pts).(Source: 5.1.8,14) An **eigenspace** is a **null space**, in this case the null space of $E - 2I$. Find the reduced row echelon form of this matrix to find the basis.

$$\begin{aligned} E - 2I &= \begin{bmatrix} 5 & -4 & 4 \\ 9 & -6 & 6 \\ -3 & 3 & -3 \end{bmatrix} \\ &\sim \begin{bmatrix} -3 & 3 & -3 \\ 9 & -6 & 6 \\ 5 & -4 & 4 \end{bmatrix} \\ &\sim \begin{bmatrix} 1 & -1 & 1 \\ 9 & -6 & 6 \\ 5 & -4 & 4 \end{bmatrix} \\ &\sim \begin{bmatrix} 1 & -1 & 1 \\ 0 & 3 & -3 \\ 0 & 1 & -1 \end{bmatrix} \\ &\sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

The general solution to $(E - 2I)\mathbf{x} = \mathbf{0}$ is

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

and so $\left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$ is a basis for the eigenspace. **(done)**

By the way, because the eigenspace is one-dimensional, any other basis for this subspace must consist of a single, nonzero multiple of $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$.