MATH 203–03 (Kunkle), Quiz 6   Name: ______________________

10 pts, 10 minutes   Oct 10, 2024

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1 (10 pts). Suppose the determinant $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 4$. Find the following determinants.

a. $\begin{vmatrix} a & b & c \\ g & h & i \\ d & e & f \end{vmatrix}$ b. $\begin{vmatrix} d & e & f \\ g & h & i \\ a & b & c \end{vmatrix}$ c. $\begin{vmatrix} -3a & -3b & -3c \\ d & e & f \\ g & h & i \end{vmatrix}$

d. $\begin{vmatrix} -3a & 3b & -3c \\ d & -e & f \\ g & -h & i \end{vmatrix}$ e. $\begin{vmatrix} a & b & c \\ d-3a & e-3b & f-3c \\ g & h & i \end{vmatrix}$

Solution:

1.(Source: 3.2.15-20, 45) The solution uses Theorems 3 and 5 of section 3.2:

a. By Thm 3, part b, $\begin{vmatrix} a & b & c \\ g & h & i \\ d & e & f \end{vmatrix} = - \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -4.$

b. This time, apply Thm 3 part b twice: $\begin{vmatrix} d & e & f \\ g & h & i \\ a & b & c \end{vmatrix} = - \begin{vmatrix} d & e & f \\ a & b & c \\ g & h & i \end{vmatrix} = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 4.$

c. Apply Thm 3 part c: $\begin{vmatrix} -3a & -3b & -3c \\ d & e & f \\ g & h & i \end{vmatrix} = -3 \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -12.$

d. See remarks following proof of Theorem 5, section 3.2: "Because of Theorem 5, each statement in Theorem 3 is true when the word *row* is replaced everywhere by the word it column." Factor -3 out of row 1, and -1 out of column 2:

$$\begin{vmatrix} -3a & 3b & -3c \\ d & -e & f \\ g & -h & i \end{vmatrix} = -3 \begin{vmatrix} a & -b & c \\ d & -e & f \\ g & -h & i \end{vmatrix} = -3(-1) \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 12.$$

e. Apply Thm 3 part a: $\begin{vmatrix} a & b & c \\ d-3a & e-3b & f-3c \\ g & h & i \end{vmatrix} = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 4.$