MATH 203–03 (Kunkle), Quiz 5 Name: ____________________
10 pts, 10 minutes Oct 1, 2024

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1 (10 pts). Suppose that the first and third rows of the $n \times n$ matrix C are identical. Explain why the **columns** of C must be linearly dependent.

1.(Source: 2.3.31) There are many correct solutions using the Invertible Matrix Theorem (theorem 8 of section 2.3). Here's one.

Since the first and third rows of C are the same, applying the row operation $\mathbf{r}3 \leftarrow \mathbf{r}3 - \mathbf{r}1$ to C creates a matrix with a row of zeros. C does not have a pivot in every row, so, by the invertible matrix theorem, C is not invertible. Since C is not invertible, by the invertible matrix theorem again, its columns must be linearly dependent. **(done)**

Here's a slightly different proof:

$$\begin{aligned} & \text{The rows of } C \text{ are linearly dependent} \\ \Rightarrow\ & \text{The columns of } C^T \text{ are linearly dependent} \\ \Rightarrow\ & C^T \text{ is singular} \\ \Rightarrow\ & C \text{ is singular} \\ \Rightarrow\ & \text{The columns of } C \text{ are linearly dependent.} \end{aligned}$$

The last three implications $\Rightarrow$ are by the Invertible Matrix Theorem. **(done)**

Note that the invertible matrix theorem applies only to square matrices. You can't prove that two identical rows imply linearly dependent columns without relying on the matrix being square, since

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$$

has two identical rows but linearly independent columns.

Tip: Memorize the *definition* of what it means for a set of vectors to be linearly independent, not just the *tests* for linear independence. See the review notes, section 1.7, at

https://kunklet.people.charleston.edu/MATH203/203review.pdf