

1 (10 pts). Find the inverse of the matrix

$$W = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

or show that it does not exist.

Solution:

1(10 pts).(Source: 2.2.41-42) Augment W with the identity and row-reduce.

row operation	result
(beginning matrix)	$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{bmatrix}$
$\mathbf{r}_2 \leftarrow \mathbf{r}_2 - \mathbf{r}_1$ $\mathbf{r}_3 \leftarrow \mathbf{r}_3 - \mathbf{r}_1$	$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -2 & -2 & -1 & 1 & 0 \\ 0 & -1 & -1 & -1 & 0 & 1 \end{bmatrix}$
$\mathbf{r}_2 \leftrightarrow \mathbf{r}_3$	$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & -1 & -1 & 0 & 1 \\ 0 & -2 & -2 & -1 & 1 & 0 \end{bmatrix}$
$\mathbf{r}_3 \leftarrow \mathbf{r}_3 - 2\mathbf{r}_2$	$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & -1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & -2 \end{bmatrix}$

Because W fails to have a pivot in its third column, it doesn't row-reduce to the identity. Therefore W is singular (that is, not invertible).