MATH 203–03 (Kunkle), Quiz 2 | Name: ____________________
10 pts, 10 minutes | Sept 3, 2024

---

1. Let $Q = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & -2 \\ 0 & -1 & 2 \\ 5 & 0 & -6 \end{bmatrix}$ and $\mathbf{p} = \begin{bmatrix} 2 \\ 0 \\ -1 \\ -1 \end{bmatrix}$.

a (2 pts). Calculate the product $Q\mathbf{p}$ or explain why it does not exist.

b (5 pts). Find the general solution to $Q\mathbf{x} = \mathbf{0}$.

c (3 pts). True or False? Justify your answer:
For every $\mathbf{b}$ in $\mathbb{R}^4$, the equation $Q\mathbf{x} = \mathbf{b}$ has exactly one solution.

Solution: 1a.(Source: 1.4.5) **Read** Definition of matrix-vector product, section 1.4 of text. $Q\mathbf{p}$ is undefined because the number of columns of $Q \neq$ the number of rows of $\mathbf{p}$.

1b.(Source: 1.4.11-12,1.5.6-12) **Read** "Homogeneous Linear Systems," section 1.5. A homogeneous equation always has the trivial solution $\mathbf{x} = \mathbf{0}$; To find any others, row reduce Q. There's no need to augment the zero column $\mathbf{0}$ if we remember that it's unchanged by row operations.

row operation	**result**
initial matrix	$\begin{matrix} 1 & 0 & -1 \\ 1 & 1 & -2 \\ 0 & -1 & 2 \\ 5 & 0 & -6 \end{matrix}$
$\mathbf{r}_2 \leftarrow \mathbf{r}_2 - \mathbf{r}_1$ $\mathbf{r}_4 \leftarrow \mathbf{r}_4 - 5\mathbf{r}_1$	$\begin{matrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & -1 & 2 \\ 0 & 0 & -1 \end{matrix}$
$\mathbf{r}_3 \leftarrow \mathbf{r}_3 + \mathbf{r}_2$	$\begin{matrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{matrix}$
$\mathbf{r}_4 \leftarrow \mathbf{r}_4 + \mathbf{r}_3$	$\begin{matrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{matrix}$

Having reached the end of the forward phase of elimination, we see that Q has a pivot in every column. A homogeneous linear system has a nontrivial solution if and only if the system has a free variable. Therefore, the only solution to $Q\mathbf{x} = \mathbf{0}$ is $x_1 = x_2 = x_3 = 0$.

1c.(Source: 1.4.17,42) False. **Read** Theorem 4, section 1.4. Because Q fails to have a pivot in every row, the columns of Q fail to span $\mathbb{R}^4$. That is, there is at least one $\mathbf{b}$ in $\mathbb{R}^4$ for which $Q\mathbf{x} = \mathbf{b}$ has no solution.

(Some students may have been misled by the fact that Q has a pivot in every column, that is, the system $Q\mathbf{x} = \mathbf{b}$ has no free variables. That implies that when $Q\mathbf{x} = \mathbf{b}$ is consistent, it must have exactly one solution.)