MATH 203–03 (Kunkle), Quiz 1 Name: ______________________________

10 pts, 10 minutes Aug 27, 2024

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1 (10 pts). Find the general solution to the system expressed here in augmented matrix form:

$$\begin{bmatrix} 1 & 0 & -2 & 2 & 7 \\ 1 & 1 & 0 & 2 & 3 \\ 2 & 1 & -2 & 5 & 15 \end{bmatrix}$$

Solution:

(Source: 1.2.12)

Forward phase:

Subtract multiples of pivot row $\mathbf{r}_1$ from rows beneath to produce zeros in column 1:

$\mathbf{r}_2 \leftarrow \mathbf{r}_2 - \mathbf{r}_1$	$\begin{matrix} 1 & 0 & -2 & 2 & 7 \\ 0 & 1 & 2 & 0 & -4 \\ 2 & 1 & -2 & 5 & 15 \end{matrix}$
$\mathbf{r}_3 \leftarrow \mathbf{r}_3 - 2\,\mathbf{r}_1$	$\begin{matrix} 1 & 0 & -2 & 2 & 7 \\ 0 & 1 & 2 & 0 & -4 \\ 0 & 1 & 2 & 1 & 1 \end{matrix}$

Subtract multiples of pivot row $\mathbf{r}_2$ from row beneath to produce zeros in column 2:

$\mathbf{r}_3 \leftarrow \mathbf{r}_3 - \mathbf{r}_2$	$\begin{matrix} 1 & 0 & -2 & 2 & 7 \\ 0 & 1 & 2 & 0 & -4 \\ 0 & 0 & 0 & 1 & 5 \end{matrix}$

End forward phase. Matrix is in row echelon form. We can already see that the system is consistent, since there's not a pivot in the last column, and that there are infinitely many solutions, since x_3 is a free variable.

Backward phase:

Subtract multiples of pivot row $\mathbf{r}_3$ from rows above to produce zeros in column 4:

$\mathbf{r}_1 \leftarrow \mathbf{r}_1 - 2\,\mathbf{r}_3$	$\begin{matrix} 1 & 0 & -2 & 0 & -3 \\ 0 & 1 & 2 & 0 & -4 \\ 0 & 0 & 0 & 1 & 5 \end{matrix}$

End backward phase. Matrix is in reduced row echelon form. You can see the calculation of this on Octave at https://sagecell.sagemath.org/?q=hysbqf.

x_3 is free, since its coefficients form a non-pivot column. General solution:

$$\begin{aligned} x_1 &= -3 + 2x_3 \\ x_2 &= -4 - 2x_3 \\ x_3 &\text{ is free} \\ x_4 &= 5 \end{aligned}$$