
1(10 pts) Evaluate the limit or show that it does not exist. One-word answers are not sufficient for full credit. Show how you arrived at your answers.

a. $\lim_{x \rightarrow 6} \frac{x-6}{x^2-36}$

b. $\lim_{x \rightarrow -5^-} \frac{1}{x+5}$

c. $\lim_{x \rightarrow \infty} \frac{64+x^3}{x^2+2x+2}$

Solution:

1a.(Source: 2.2.16) Factor the difference of squares and cancel the common factor:

$$\lim_{x \rightarrow 6} \frac{x-6}{x^2-36} = \lim_{x \rightarrow 6} \frac{x-6}{(x+6)(x-6)} = \lim_{x \rightarrow 6} \frac{1}{x+6} = \frac{1}{12}$$

1b.(Source: 2.1.51) " $\frac{1}{0}$ " indicates that the limit is one of $\pm\infty$. To decide which, determine the sign of $\frac{1}{x+5}$ when x is the left of -5 :

$$x < -5 \implies x+5 < 0 \implies \frac{1}{x+5} < 0,$$

so the limit must be $-\infty$.

1c.(Source: 2.3.14) As $x \rightarrow \infty$, the limit is determined by the lead terms in the top and bottom:

$$\lim_{x \rightarrow \infty} \frac{64+x^3}{x^2+2x+2} = \lim_{x \rightarrow \infty} \frac{x^3}{x^2}$$

Cancel the common factor:

$$= \lim_{x \rightarrow \infty} x = \infty.$$