
1(10 pts). Let

$$f(x) = \sqrt{2-x} \qquad g(x) = \frac{3}{x^2+4} \qquad h(x) = \begin{cases} x+1 & \text{if } 1 \leq x \\ 1-2x & \text{if } -1 \leq x < 1 \end{cases}$$

Find the following.

- a. The domain of $f(x)$.
- b. The domain of $g(x)$.
- c. The domain of $h(x)$.
- d. The function $g(f(x))$.

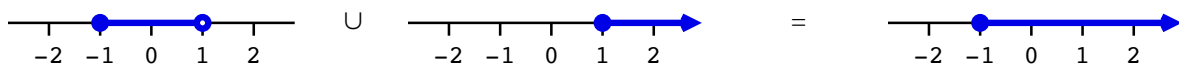
Simplify your answer to part d.

Solution:

1a.(Source: 1.1) The square root is defined only if $0 \leq 2-x$, or $x \leq 2$. The domain is $(-\infty, 2]$.

1b.(Source: 1.1) $x^2 + 4 = 0$ would imply $x^2 = -4$. This equation has no real solutions, so the fraction $g(x)$ is defined for all real numbers x . The domain is $(-\infty, \infty)$.

1c.(Source: 1.1) This piecewise defined function is defined to be $x+1$ for all x in $[1, \infty)$ and $1-2x$ for all x in $[-1, 1)$. The domain is their union, $[-1, 1) \cup [1, \infty)$, or, more simply, $[-1, \infty)$. Here's a picture:



1d.(Source: 1.2) We can express the function g as:

$$g(\quad) = \frac{3}{(\quad)^2 + 4}.$$

Then

$$g(f(x)) = g(\sqrt{2-x}) = \frac{3}{(\sqrt{2-x})^2 + 4} = \frac{3}{2-x+4} = \frac{3}{6-x}.$$

(done)