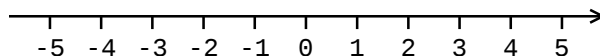


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Read each problem carefully and simplify your answers.

Unless otherwise indicated, supporting work will be required on every problem worth more than 2 points.

1a(2 pts). Sketch the set $(-3, \infty)$ on the number line.

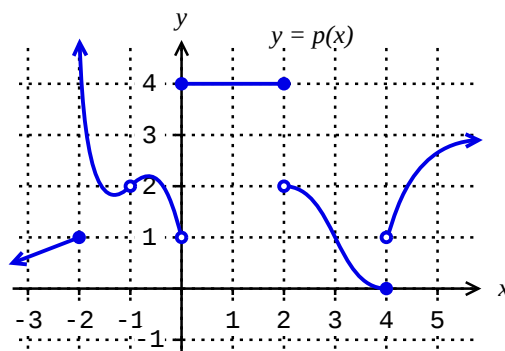


1b(2 pts). Write the set pictured here in interval form:



2(8 pts). Use the graph of $p(x)$ to find the following. Supporting work is not required. Your answer to a-h should be ∞ , $-\infty$, a number, or “dne.”

- | | |
|-------------------------------------|------------------------------------|
| a. $p(-1)$ | b. $p(1)$ |
| c. $\lim_{x \rightarrow 3} p(x)$ | d. $\lim_{x \rightarrow 4} p(x)$ |
| e. $\lim_{x \rightarrow 0^-} p(x)$ | f. $\lim_{x \rightarrow 0^+} p(x)$ |
| g. $\lim_{x \rightarrow -2^+} p(x)$ | h. $p(-2)$ |

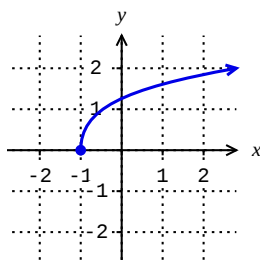


3(2 pts) At what x -values, if any, is the function $p(x)$ in problem 2 **discontinuous**?

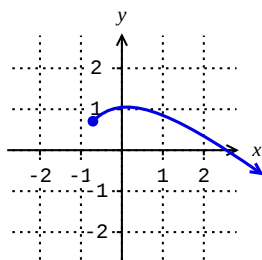
4(16 pts). Find all solutions x to each of the given equations.

- | | | |
|-------------------------------|--------------------|--|
| a. $2(x - 2) + 3 = 5(3 - x)$ | b. $ x = 13$ | c. $\frac{3}{2x} + \frac{1}{5} = \frac{4}{5x}$ |
| d. $8x^2 + 5x + 3 = 2x^2 + 7$ | e. $(x + 4)^2 = 9$ | |

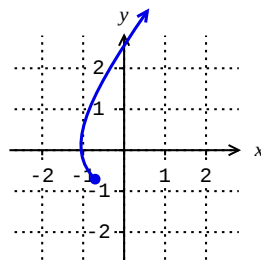
5(4 pts). Determine whether the given curve is the graph of a function $y = f(x)$.



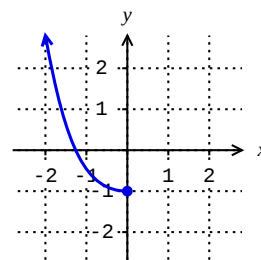
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Function
Not a function



(circle one)
Function
Not a function



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Function
Not a function



(circle one)
Function
Not a function

6(3 pts). Find the average rate of change of the function $r(x) = \frac{2}{3x-1}$ between $x_1 = \frac{1}{2}$ and $x_2 = 1$.

7(8 pts). Expand the polynomial. That is, rewrite the polynomial without parentheses.

a. $(2x - 5)(5x + 2)$

b. $(x - 3)^2$

8(4 pts). Simplify the expression $\frac{x^{-2} \cdot x^{3/4}}{(x^2)^3}$. Write your answer so that the variable doesn't appear in the denominator.

9(8 pts). Let $g(x) = \frac{x-4}{2x}$ and $h(x) = \frac{1}{x}$. Find and simplify the following.

a. $h(g(x))$

b. $h(x)g(x)$

c. $\frac{g(x)}{h(x)}$

10(8 pts). Find an equation of the line described in each part.

a. The line passing through the points $(-2, 3)$ and $(10, 0)$.

b. The line passing through the origin $(0, 0)$ that is perpendicular to the line $5x - 2y = 7$.

11(9 pts). A baker finds that when she prices her cakes at \$40 each, she sells 150 cakes. When she prices her cakes at \$60, she sells only 90 cakes.

a. Find her demand function $D(x)$, assuming it is linear.

b. What is her revenue function?

12(6 pts). A puppet-maker sells his puppets for 100 dollars each. To produce x puppets in a month costs him $C(x) = 1620 + 80x$ dollars. What number of puppets is his break-even point?

13(20 pts). Evaluate the given limit or explain why it does not exist.

a. $\lim_{x \rightarrow 2^-} \frac{x-5}{x-2}$

b. $\lim_{x \rightarrow 2^+} \begin{cases} x^2 + 6 & \text{if } x < 2 \\ 3 - 2x & \text{if } 2 \leq x \end{cases}$

c. $\lim_{x \rightarrow 2} \frac{x-2}{x^2 - 7x + 10}$

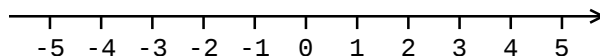
d. $\lim_{x \rightarrow -\infty} \frac{2x^2 + 9x}{3x + 4}$

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c. $\lim_{x \rightarrow 0^-} p(x)$

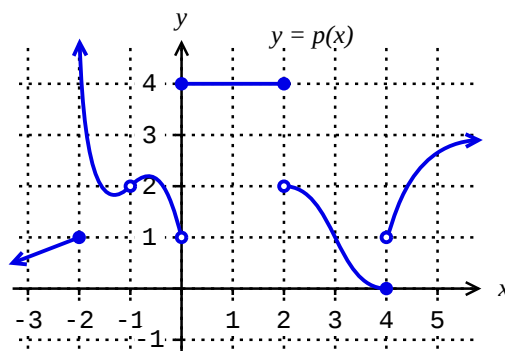
d. $\lim_{x \rightarrow 0^+} p(x)$

e. $\lim_{x \rightarrow 3} p(x)$

f. $\lim_{x \rightarrow 4} p(x)$

g. $p(-1)$

h. $p(1)$



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4(16 pts). Find all solutions x to each of the given equations.

a. $2(x - 2) + 3 = 7(3 - x)$

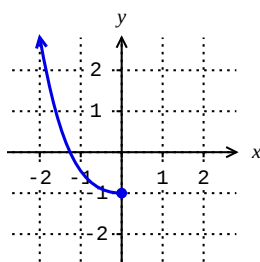
b. $|x| = -12$

c. $\frac{4}{2x} + \frac{1}{5} = \frac{3}{5x}$

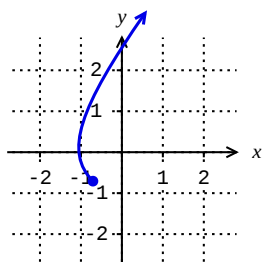
d. $8x^2 - 5x + 3 = 2x^2 + 7$

e. $(x - 3)^2 = 16$

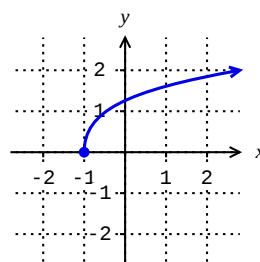
5(4 pts). Determine whether the given curve is the graph of a function $y = f(x)$.



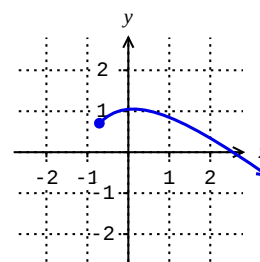
(circle one)
Function
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6(3 pts). Find the average rate of change of the function $r(x) = \frac{2}{3x-1}$ between $x_1 = \frac{1}{2}$ and $x_2 = 1$.

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a. $(2x + 5)(5x - 2)$

b. $(x - 2)^2$

8(4 pts). Simplify the expression $\frac{x^{-2} \cdot x^{3/4}}{(x^2)^3}$. Write your answer so that the variable doesn't appear in the denominator.

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a. $h(g(x))$

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a. The line passing through the points $(-2, 4)$ and $(10, 0)$.

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11(9 pts). A baker finds that when she prices her cakes at \$40 each, she sells 120 cakes. When she prices her cakes at \$60, she sells only 60 cakes.

a. Find her demand function $D(x)$, assuming it is linear.

b. What is her revenue function?

12(6 pts). A puppet-maker sells his puppets for 100 dollars each. To produce x puppets in a month costs him $C(x) = 1480 + 80x$ dollars. What number of puppets is his break-even point?

13(20 pts). Evaluate the given limit or explain why it does not exist.

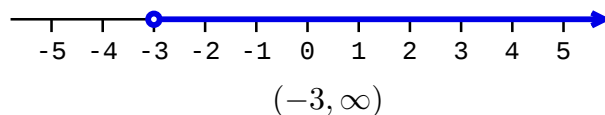
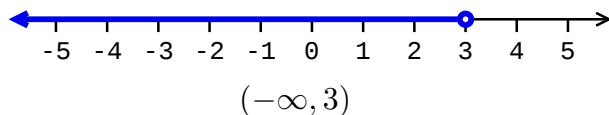
a. $\lim_{x \rightarrow 2^+} \frac{x-5}{x-2}$

b. $\lim_{x \rightarrow 2^-} \begin{cases} x^2 + 5 & \text{if } x < 2 \\ 3 - 2x & \text{if } 2 \leq x \end{cases}$

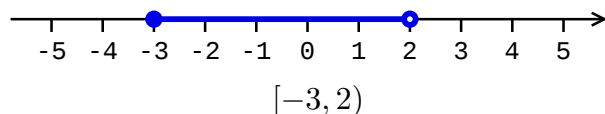
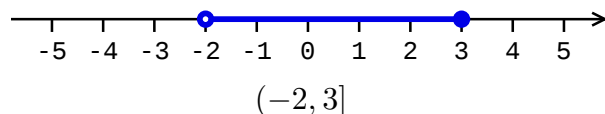
c. $\lim_{x \rightarrow 2} \frac{x-2}{x^2 - 5x + 6}$

d. $\lim_{x \rightarrow -\infty} \frac{2x^2 + 9x}{3x^2 + 4}$

1a.(Source: 0.1) There were two forms of this question (and most questions on the exam). Here are both answers.



1b.(Source: 0.1)



2.(Source: 2.2.4-9, 2.6.3-4) (order varies.)

$$\lim_{x \rightarrow -2^+} p(x) = \infty$$

$$\lim_{x \rightarrow 0^-} p(x) = 1$$

$$\lim_{x \rightarrow 4} p(x) \text{ does not exist.}$$

$$p(1) = 4$$

$$p(-2) = 1$$

$$p(-1) \text{ does not exist.}$$

$$\lim_{x \rightarrow 0^+} p(x) = 4$$

$$\lim_{x \rightarrow 3} p(x) = 1$$

3.(Source: 2.4) $p(x)$ is discontinuous at $x = -2, 0, 2$, and 4 , where the limit does not exist. p is also discontinuous at $x = -1$, where the limit exists but the function does not.

4a.(Source: 0.6.93)

Alternate form:

$$2(x - 2) + 3 = 5(3 - x)$$

$$2x - 4 + 3 = 15 - 5x$$

$$2x + 5x = 15 + 4 - 3$$

$$7x = 16$$

$$x = \frac{16}{7}$$

$$2(x - 2) + 3 = 7(3 - x)$$

$$2x - 4 + 3 = 21 - 7x$$

$$2x + 7x = 21 + 4 - 3$$

$$9x = 22$$

$$x = \frac{22}{9}$$

4b.(Source: 0.1.27,31) $|x| = 13$ implies $x = \pm 13$. Alternate form: The absolute value cannot be negative, so $|x| = -12$ has no solutions.

(Source: 0.8.17) 4c.(Source: 0.8.17) We can rewrite the equation with no fractions if we multiply both sides by $10x$:

Alternate form:

$$10x \left(\frac{3}{2x} + \frac{1}{5} \right) = \frac{4}{5x} \cdot 10x$$

$$10x \frac{3}{2x} + 10x \frac{1}{5} = \frac{4}{5x} \cdot 10x$$

$$5 \cdot 3 + 2x = 4 \cdot 2$$

$$15 + 2x = 8$$

$$2x = 8 - 15 = -7$$

$$x = -\frac{7}{2}$$

$$10x \left(\frac{4}{2x} + \frac{1}{5} \right) = \frac{3}{5x} \cdot 10x$$

$$10x \frac{4}{2x} + 10x \frac{1}{5} = \frac{3}{5x} \cdot 10x$$

$$5 \cdot 4 + 2x = 3 \cdot 2$$

$$20 + 2x = 6$$

$$2x = 6 - 20 = -14$$

$$x = -7$$

4d.(Source: 0.7.8) Get a zero on one side and factor the other.

$$8x^2 + 5x + 3 = 2x^2 + 7 \implies 0 = 6x^2 + 5x - 4 = (3x + 4)(2x - 1) \implies x = -\frac{4}{3} \text{ or } \frac{1}{2}$$

Alternate form:

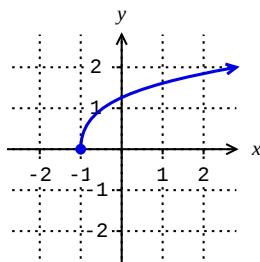
$$8x^2 - 5x + 3 = 2x^2 + 7 \implies 0 = 6x^2 - 5x - 4 = (3x - 4)(2x + 1) \implies x = \frac{4}{3} \text{ or } -\frac{1}{2}$$

4e.(Source: 0.7.13) $x + 4 = \pm 3$, so $x = -4 \pm 3 = -7$ or -1 .

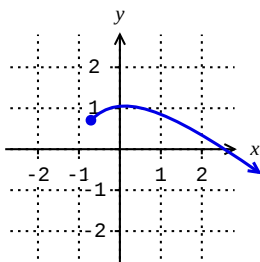
Alternate form: $x - 3 = \pm 4$, so $x = 3 \pm 4 = 7$ or -1 .

5.(Source: 1.1.40-51) These graphs appear in different order on different exams.

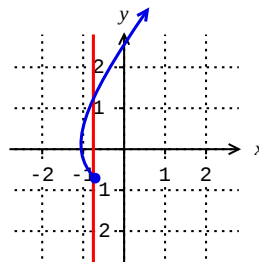
Vertical Line Test: A curve is the graph of a function $y = f(x)$ if and only if no vertical line intersects the curve more than once.



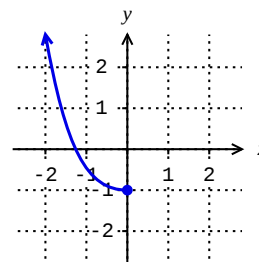
Function



Function



Not a function



Function

6.(Source: 2.5.10) $r(1) = \frac{2}{3-1} = 1$, and $r(\frac{1}{2}) = \frac{2}{\frac{3}{2}-1} = \frac{2}{\frac{1}{2}} = 4$. Average rate of change is

$$\frac{r(1) - r(\frac{1}{2})}{1 - \frac{1}{2}} = \frac{1 - 4}{\frac{1}{2}} = -6.$$

7a.(Source: 0.4.21) FOIL: $(2x - 5)(5x + 2) = 10x^2 + 4x - 25x - 10 = 10x^2 - 21x - 10$.

Alternate form: $(2x + 5)(5x - 2) = 10x^2 - 4x + 25x - 10 = 10x^2 + 21x - 10$.

7b.(Source: 0.4.24) FOIL, or use $(a - b)^2 = a^2 - 2ab + b^2$.

$(x - 3)^2 = x^2 - 6x + 9$. Alternate form: $(x - 2)^2 = x^2 - 4x + 4$.

8.(Source: 0.3.12) $\frac{x^{-2} \cdot x^{3/4}}{(x^2)^3} = \frac{x^{-2+3/4}}{x^{2 \cdot 3}} = \frac{x^{-5/4}}{x^6} = x^{-5/4-6} = x^{-29/4}$.

9a.(Source: 1.2.21) $h(g(x)) = \frac{1}{g(x)} = \frac{1}{\frac{x-4}{2x}} = \frac{2x}{x-4}$.

Alternate form: $h(g(x)) = \frac{1}{g(x)} = \frac{1}{\frac{2x}{x-4}} = \frac{x-4}{2x}$.

9b.(Source: 1.2.6) $h(x)g(x) = \frac{1}{x} \cdot \frac{x-4}{2x} = \frac{x-4}{2x^2}$.

Alternate form: $h(x)g(x) = \frac{1}{x} \cdot \frac{2x}{x-4} = \frac{2}{x-4}$.

9c.(Source: 1.2.8) $\frac{g(x)}{h(x)} = \frac{\frac{1}{x}}{\frac{x-4}{2x}} = \frac{1}{x} \cdot \frac{2x}{x-4} = \frac{2}{x-4}$.

Alternate form: $\frac{h(x)}{g(x)} = \frac{\frac{2x}{x-4}}{\frac{1}{x}} = \frac{2x}{x-4} \cdot x = \frac{2x^2}{x-4}.$

10a.(Source: 0.5.13) The slope of the line is $\frac{\Delta y}{\Delta x} = \frac{3-0}{-2-10} = -\frac{1}{4}.$ The line can be written in point-slope form as $y = -\frac{1}{4}(x-10).$

Alternate form: Slope is $\frac{4-0}{-2-10} = -\frac{1}{3},$ and line is $y = -\frac{1}{3}(x-10).$

10b.(Source: 0.5.39) Rewrite the line $5x - 2y = 7$ as $y = \frac{5}{2}x - \frac{7}{2},$ so its slope is $\frac{5}{2}.$ Any line perpendicular to this has slope $-\frac{2}{5}.$ Since the desired line passes through $(0,0),$ its equation is $y = -\frac{2}{5}x.$

Alternate form: Any line parallel to $y = \frac{5}{2}x - \frac{7}{2}$ has slope $\frac{5}{2}.$ Since the desired line passes through $(0,0),$ its equation is $y = \frac{5}{2}x.$

11a.(Source: 1.3.37) $D(x)$ is the (greatest) price at which the baker can expect to sell x cakes. To say that $D(x)$ is linear means it is a function of the form $D(x) = mx + b.$ The slope m equals

$$\frac{\Delta D}{\Delta x} = \frac{\Delta \text{ price}}{\Delta \text{ quantity}} = \frac{\$40 - \$60}{150 - 90} = \frac{-20}{60} = -\frac{1}{3}.$$

Now write the demand line in point-slope form and solve for $D:$

$$D(x) - 40 = -\frac{1}{3}(x - 150) = -\frac{1}{3}x + 50 \implies D(x) = -\frac{1}{3}x + 90.$$

The most common error on this problem was to use $x = \text{price}$ and $D(x) = \text{quantity}.$ It should be $x = \text{quantity}$ and $D(x) = \text{price}.$

11b. Revenue equals price times quantity: $R(x) = xD(x) = -\frac{1}{3}x^2 + 90x.$

Alternate form: 11a. Slope $m =$

$$\frac{\Delta D}{\Delta x} = \frac{\Delta \text{ price}}{\Delta \text{ quantity}} = \frac{\$40 - \$60}{120 - 60} = \frac{-20}{60} = -\frac{1}{3},$$

and

$$D(x) - 40 = -\frac{1}{3}(x - 120) = -\frac{1}{3}x + 40 \implies D(x) = -\frac{1}{3}x + 80.$$

11b. $R(x) = xD(x) = -\frac{1}{3}x^2 + 80x.$

12.(Source: 1.3.29-32) Revenue equals price times quantity $= 100x$ and profit equals revenue minus cost $= 100x - (1620 + 80x) = 20x - 1620.$ The break-even point is the x -value where profit equals zero:

$$20x - 1620 = 0 \implies x = \frac{1620}{20} = 81.$$

Alternate form: Profit $= 100x - (1480 + 80x) = 20x - 1480 = 0$ at $x = \frac{1480}{20} = 74.$

13a.(Source: 2.1.49) $\lim_{x \rightarrow 2^-} \frac{x-5}{x-2} = \frac{-3}{0}$, indicating an infinite limit. To decide whether this limit is ∞ or $-\infty$, determine the sign of $\frac{x-5}{x-2}$:

$$x \rightarrow 2^- \implies x < 2 \implies x - 2 < 0 \implies \frac{x-5}{x-2} = \frac{-}{-} = +$$

Therefore $\lim_{x \rightarrow 2^-} \frac{x-5}{x-2}$ must be $+\infty$.

Alternate form:

$$x \rightarrow 2^+ \implies x > 2 \implies x - 2 > 0 \implies \frac{x-5}{x-2} = \frac{-}{+} = -$$

Therefore $\lim_{x \rightarrow 2^+} \frac{x-5}{x-2}$ must be $-\infty$.

13b.(Source: 2.1.54-59) The function in question equals $3 - 2x$ when $x > 2$. Therefore

$$\lim_{x \rightarrow 2^+} \begin{cases} x^2 + 6 & \text{if } x < 2 \\ 3 - 2x & \text{if } 2 \leq x \end{cases} = \lim_{x \rightarrow 2^+} (3 - 2x) = 3 - 4 = -1.$$

Alternate form: The function in question equals $x^2 + 5$ when $x < 2$. Therefore

$$\lim_{x \rightarrow 2^-} \begin{cases} x^2 + 5 & \text{if } x < 2 \\ 3 - 2x & \text{if } 2 \leq x \end{cases} = \lim_{x \rightarrow 2^-} (x^2 + 5) = 2^2 + 5 = 9.$$

13c.(Source: 2.2.14, 2.3.15-16) $\lim_{x \rightarrow 2} \frac{x-2}{x^2-7x+10} = \frac{0}{0}$, an indeterminate form. To evaluate the limit, cancel the common factor:

$$\lim_{x \rightarrow 2} \frac{x-2}{x^2-7x+10} = \lim_{x \rightarrow 2} \frac{x-2}{(x-2)(x-5)} = \lim_{x \rightarrow 2} \frac{1}{x-5} = -\frac{1}{3}.$$

Alternate form:

$$\lim_{x \rightarrow 2} \frac{x-2}{x^2-5x+6} = \lim_{x \rightarrow 2} \frac{x-2}{(x-2)(x-3)} = \lim_{x \rightarrow 2} \frac{1}{x-3} = -1.$$

13d.(Source: 2.3.10,12) As $x \rightarrow -\infty$, the limit is determined entirely by the lead terms in the top and bottom:

$$\lim_{x \rightarrow -\infty} \frac{2x^2 + 9x}{3x + 4} = \lim_{x \rightarrow -\infty} \frac{2x^2}{3x} = \lim_{x \rightarrow -\infty} \frac{2x}{3} = -\infty.$$

Alternate form:

$$\lim_{x \rightarrow -\infty} \frac{2x^2 + 9x}{3x^2 + 4} = \lim_{x \rightarrow -\infty} \frac{2x^2}{3x^2} = \lim_{x \rightarrow -\infty} \frac{2}{3} = \frac{2}{3}.$$