Name: ______________________________

You may use an approved calculator, but no notes, books, phones, smart watches, earbuds, other electronic devices, or any other outside material. Notes stored in your calculator in the form of programs, equations, etc., are not allowed. You may not share a calculator with another student. Do not leave the room during the exam with a phone.

Read each problem carefully and simplify your answers.

Unless otherwise indicated, supporting work will be required on every problem worth more than 2 points.

1(8 pts). Find the second derivative of the given function.

a. $\ell(x) = \dfrac{1}{(3x+1)^2}$ b. $m(x) = x^4 - 2x^2$

2(6 pts). Find the x-coordinates of the inflection points of $m(x) = x^4 - 2x^2$. (The y-coordinates are not required.)

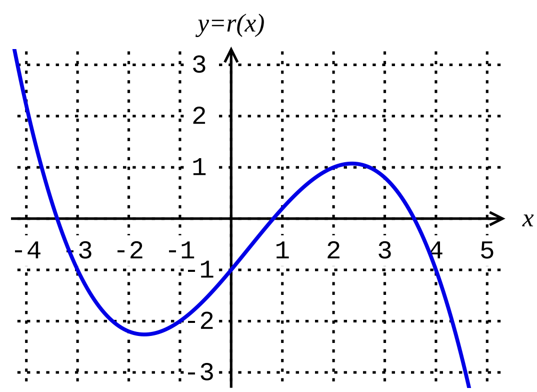
3(12 pts). For each function, give the equations of all vertical and horizontal asymptotes (if they exist) of the graph. Supporting work not required.

a. $f(x) = \dfrac{5x}{7x-2}$ b. $g(x) = \dfrac{5}{7x-2}$

4(3 pts). Suppose $q(x)$ is a function for which $q'(3) = 0$ and $q''(3) = -2$. Must $q(x)$ have a local max or a local min at $x = 3$?

(circle one): local maximum local minimum Not enough information.

5. See the graph of $r(x)$ provided.



a(2 pts). Estimate the interval(s) of the x-axis on which the graph of $r(x)$ is concave up.

b(2 pts). Estimate the interval(s) of the x-axis on which the graph of $r(x)$ is concave down.

6(24 pts). Find the derivative of the given function. You are not required to simplify your answers to this question.

a. $\ln(5x^2 - 3x + 6)$ b. $\sqrt{\ln(x+1)}$ c. $\dfrac{e^x}{2e^x+1}$ d. $e^{(1-2x^3)}$

7(6 pts). Expand the logarithmic expression as much as possible into the sum and/or difference of logarithmic expressions. No term should contain exponents.

$$\ln\left(5x^3y^4\right)$$

8(15 pts). Solve for x in the given equation.

a. $3e^{4x} = 60$ b. $7^{x-2} = 4$ c. $\ln(x^2 + 2x) - \ln(x) = \ln(12)$

9(10 pts). A baker finds that when she prices her cakes at \$40 each, she sells 120 cakes. When she prices her cakes at \$60, she sells only 60 cakes.

a. Find her demand function $D(x)$, assuming it is linear.

b. What is her revenue function?

c. Find her marginal revenue function.

The future value F of P dollars invested at an annual rate of r for t years.

compounded n times per year: compounded continuously:

$$F = P\left(1 + \frac{r}{n}\right)^{nt} \qquad\qquad F = Pe^{rt}$$

10(12 pts). An investment account in Bank A pays 3.0% compounded monthly, and an account in Bank B pays 2.5% compounded continuously. I need to have \$1000.00 in 5 years. How much would I have to depost today in each bank to reach my goal? In which bank would you advise me to place my investment?

1a(8 pts).(Source: 4.4) $\ell(x) = (4x+1)^{-2}$. By the chain rule, $\ell'(x) = -2(4x+1)^{-3} \cdot 4$, and $\ell''(x) = (-2)(-3)(4x+1)^{-4} \cdot 4 \cdot 4 = 96(4x+1)^{-4}$.
Alternate form: $\ell''(x) = (-2)(-3)(3x+1)^{-4} \cdot 3 \cdot 3 = 54(3x+1)^{-4}$.

1b(8 pts).(Source: 4.4) $m'(x) = 4x^3 - 3 \cdot 2x$ and so $m''(x) = 4 \cdot 3x^2 - 3 \cdot 2 = 12x^2 - 6$.
Alternate form: $m''(x) = 4 \cdot 3x^2 - 2 \cdot 2 = 12x^2 - 4$.

2(6 pts).(Source: 4.4) Solve $m''(x) = 0$:

$$12x^2 - 6 = 0 \quad \Longrightarrow \quad 12x^2 = 6 \quad \Longrightarrow \quad x^2 = \frac{6}{12} = \frac{1}{2} \quad \Longrightarrow \quad x = \pm\sqrt{\frac{1}{2}}.$$

The graph of $m(x)$ at https://www.desmos.com/calculator/nt01bnoj5o confirms that both of these are inflection points, that is, points where the concavity of the graph of $m(x)$ changes.
Alternate form:

$$12x^2 - 4 = 0 \quad \Longrightarrow \quad 12x^2 = 4 \quad \Longrightarrow \quad x^2 = \frac{4}{12} = \frac{1}{3} \quad \Longrightarrow \quad x = \pm\sqrt{\frac{1}{3}}.$$

The graph of $m(x)$ at https://www.desmos.com/calculator/o8krjteghu confirms that both of these are inflection points.

3.(Source: 4.4) Horizontal Asymptotes:

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{3x}{7x} = \frac{3}{7},$$

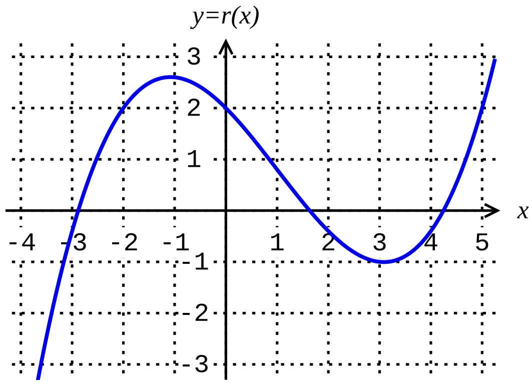
so $y = \frac{3}{7}$ is a horizontal asymptote of $f(x)$. (4 pts)

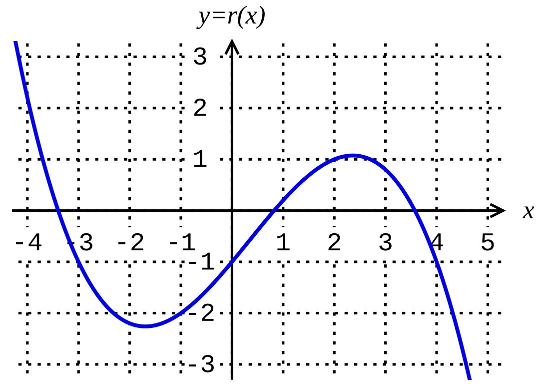
$$\lim_{x\to\infty} g(x) = \lim_{x\to\infty} \frac{3}{7x} = \text{“}\frac{1}{\infty}\text{”} = 0,$$

so $y = 0$ is a horizontal asymptote of $g(x)$. (4 pts)
Vertical asymptotes: As $x \to \frac{4}{7}$ both $f(x)$ and $g(x)$ go to “$\frac{\text{nonzero}}{0}$” $= \pm\infty$, so $x = \frac{4}{7}$ is a vertical asymptote of both $f(x)$ and $g(x)$. (4 pts)
Alternate form: $y = \frac{5}{7}$ is a horizontal asymptote of $f(x)$, $y = 0$ is a horizontal asymptote of $g(x)$, and $x = \frac{2}{7}$ is a vertical asymptote of both $f(x)$ and $g(x)$.

4(3 pts).(Source: 4.2) Since $q'(3) = 0$ and $q''(3) > 0$, $q(3)$ is a local min, by Second Derivative Test. See page 67 of the review notes, at
https://kunklet.people.charleston.edu/MATH116/Z116review.pdf.
For example, graph the function $q(x) = 1+(x-3)^2$, which satisfies $q'(3) = 0$ and $q''(3) = 2$.
Alternate form: Since $q'(3) = 0$ and $q''(3) < 0$, $q(3)$ is a local max, by Second Derivative Test. For example, graph the function $q(x) = 1 - (x-3)^2$, which satisfies $q'(3) = 0$ and $q''(3) = -2$.

5.(Source: 4.1) The function $r(x)$, graphed below left, is concave up on $(1,\infty)$ and concave down on $(-\infty,1)$





Alternate form: The function $r(x)$, graphed above right, is concave up on $(-\infty,0)$ and concave down on $(0,\infty)$.

6a(5 pts).(Source: 5.3) Chain rule. (See page 83 of the review notes.)

$$\big(\ln(3x^2+7x-6)\big)' = \frac{1}{3x^2+7x-6}(3x^2+7x-6)' = \frac{6x+7}{3x^2+7x-6}$$

Alternate form:

$$\big(\ln(5x^2-3x+6)\big)' = \frac{1}{5x^2-3x+6}(5x^2-3x+6)' = \frac{10x-3}{5x^2-3x+6}$$

6b(6 pts).(Source: 5.3) Chain rule:

$$\begin{aligned}\big((\ln(x+1))^{1/2}\big)' = \tfrac{1}{2}(\ln(x+1))^{-1/2}(\ln(x+1))' &= \tfrac{1}{2}(\ln(x+1))^{-1/2}\frac{1}{x+1}(x+1)' \\ &= \tfrac{1}{2}(\ln(x+1))^{-1/2}\frac{1}{x+1}\end{aligned}$$

6c(8 pts).(Source: 5.4) Quotient rule:

$$\left(\frac{e^x}{2e^x+1}\right)' = \frac{(e^x)'(2e^x+1)-e^x(2e^x+1)'}{(2e^x+1)^2} = \frac{e^x(2e^x+1)-e^x\cdot 2e^x}{(2e^x+1)^2} = \frac{e^x}{(2e^x+1)^2}.$$

6d(5 pts).(Source: 5.4) Chain Rule. (See page 85 of the review notes.)

$$\left(e^{(1-2x^4)}\right)' = \left(e^{(1-2x^4)}\right)(1-2x^4)' = -8x^3e^{(1-2x^4)}$$

Alternate form:

$$\left(e^{(1-2x^3)}\right)' = \left(e^{(1-2x^3)}\right)(1-2x^3)' = -6x^2e^{(1-2x^3)}$$

7(6 pts).(Source: 5.2) See page 79 of the review notes. By rule 5,

$$\ln(7x^2y^3) = \ln(7)+\ln(x^2)+\ln(y^3),$$

and by rule 7, this equals

$$= \ln(7) + 2\ln(x) + 3\ln(y)$$

.

Alternate form:

$$\ln(5x^3y^4) = \ln(5) + \ln(x^3) + \ln(y^4) = \ln(5) + 3\ln(x) + 4\ln(y)$$

8a(4 pts).(Source: 5.2) This problem uses the property $\ln(e^x) = x$. Divide by 3, take the ln of boh sides, and divide by 4:

$$e^{4x} = 15 \quad\Longrightarrow\quad \ln(e^{4x}) = 4x = \ln(15) \quad\Longrightarrow\quad x = \frac{1}{4}\ln(15)$$

Alternate form:

$$e^{4x} = 20 \quad\Longrightarrow\quad \ln(e^{4x}) = 4x = \ln(20) \quad\Longrightarrow\quad x = \frac{1}{4}\ln(20)$$

8b(5 pts).(Source: 5.2) This problem uses the property $\ln(A^x) = x\ln A$. Take ln of both sides, divide by $\ln 7$, and then add 2:

$$\ln(7^{x-2}) = (x-2)\ln(7) = \ln(3) \quad\Longrightarrow\quad x - 2 = \frac{\ln 3}{\ln 7} \quad\Longrightarrow\quad x = 2 + \frac{\ln 3}{\ln 7}$$

Alternate form:

$$\ln(7^{x-2}) = (x-2)\ln(7) = \ln(4) \quad\Longrightarrow\quad x - 2 = \frac{\ln 4}{\ln 7} \quad\Longrightarrow\quad x = 2 + \frac{\ln 4}{\ln 7}$$

8c(6 pts).(Source: 5.2) Combine terms on the right side using $\ln(A/B) = \ln A - \ln B$, cancel the common factor, then raise e to both sides.

$$\begin{aligned} \ln\left(\frac{x^2+2x}{x}\right) = \ln(x+2) &= \ln(16) \\ e^{\ln(x+2)} &= e^{\ln(16)} \\ x+2 &= 16 \\ x &= 14 \end{aligned}$$

Alternate form:

$$\begin{aligned} \ln\left(\frac{x^2+2x}{x}\right) = \ln(x+2) &= \ln(12) \\ e^{\ln(x+2)} &= e^{\ln(12)} \\ x+2 &= 12 \\ x &= 10 \end{aligned}$$

9(Source: 4.5)

a(7 pts). Recall that $D(x)$ is the (greatest) price at which the baker canexpect to sell x cakes. To say that $D(x)$ is linear means it is a function of the form $D(x) = mx + b$. The slope m equals

$$\frac{\Delta D}{\Delta x} = \frac{\Delta \text{ price}}{\Delta \text{quantity}} = \frac{\$40 - \$60}{150 - 90} = \frac{-20}{60} = -\frac{1}{3}.$$

Now write the demand line in point-slope form and solve for D:

$$D(x) - 40 = -\frac{1}{3}(x - 150) = -\frac{1}{3}x + 50 \quad \Longrightarrow \quad D(x) = -\frac{1}{3}x + 90.$$

The most common error on this problem was to use x = price and $D(x)$ = quantity. It should be x = quantity and $D(x)$ = price.

9b(2 pts). Revenue equals price times quantity: $R(x) = xD(x) = -\frac{1}{3}x^2 + 90x$.

9c(1 pts). Marginal revenue is $R'(x) = -\frac{2}{3}x + 90$.

Alternate form: 9a. Slope $m =$

$$\frac{\Delta D}{\Delta x} = \frac{\Delta \text{ price}}{\Delta \text{quantity}} = \frac{\$40 - \$60}{120 - 60} = \frac{-20}{60} = -\frac{1}{3},$$

and

$$D(x) - 40 = -\frac{1}{3}(x - 120) = -\frac{1}{3}x + 40 \quad \Longrightarrow \quad D(x) = -\frac{1}{3}x + +80.$$

9b(2 pts). $R(x) = xD(x) = -\frac{1}{3}x^2 + +80x$.

9c(1 pts). $R'(x) = -\frac{2}{3}x + +80$.

10(12 pts).(Source: 5.1) Given $F = \$2000.00$, solve for P in the equations

Bank A

$$P\left(1 + \frac{0.03}{12}\right)^{12 \cdot 5} = \$2000.00$$

$$P = \frac{\$2000.00}{(1.0025)^{60}} = \$1,721.74$$

Bank B

$$Pe^{.025 \cdot 5} = \$2000.00$$

$$P = \frac{\$2000.00}{e^{.125}} = \$1,764.99$$

Bank A provides the best deal for an investor, since the smaller investment there yields \$2000 in five years.

Alternate form:

Bank A

$$P\left(1 + \frac{0.03}{12}\right)^{12 \cdot 5} = \$1000.00$$

$$P = \frac{\$1000.00}{(1.0025)^{60}} = \$860.87$$

Bank B

$$Pe^{.025 \cdot 5} = \$1000.00$$

$$P = \frac{\$1000.00}{e^{.125}} = \$882.50$$

Bank A provides the best deal for an investor

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