MATH 116–5 (Kunkle), Exam 2 Name: ______________________________

100 pts, 75 minutes March 17, 2026

You may use an approved calculator, but no notes, books, phones, smart watches, earbuds, other electronic devices, or any other outside material. Notes stored in your calculator in the form of programs, equations, etc., are not allowed. You may not share a calculator with another student. Do not leave the room during the exam with a phone.
Read each problem carefully and simplify your answers.
Unless otherwise indicated, supporting work will be required on every problem worth more than 2 points.

1(4 pts). Find the x-values, if any, at which the given function is discontinuous.

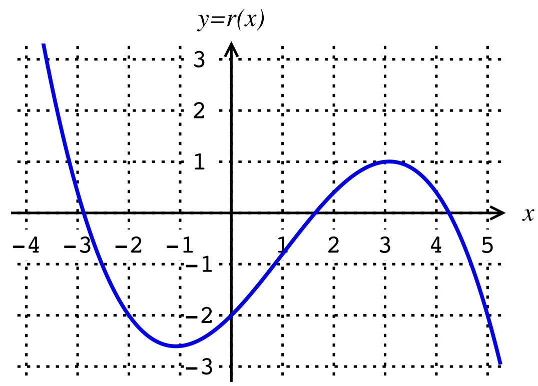
a. $f(x) = \dfrac{7x}{3x-2}$

b. $g(x) = \begin{cases} 2x+1 & \text{if } x \le 1 \\ 3-x & \text{if } x > 1 \end{cases}$

2. See the graph of $r(x)$ provided.

a(6 pts). Calculate the average rate of change of $r(x)$ from $x = -2$ to $x = 3$. Show me where your answer comes from.

b(6 pts). Estimate the instantaneous rate of change of $r(x)$ at $x = 4$. Show me where your answer comes from.



3(32 pts). Find the derivative of the given function. You are not required to simplify your answers to this question.

a. $\dfrac{1}{16}x^4 - 6\sqrt{x} + 25$

b. $(6x^2-1)^5$

c. $4x(6x^2-1)^5$

d. $\dfrac{4x-2}{4x^2+x}$

e. $\dfrac{7x+2}{\sqrt{5x+1}}$

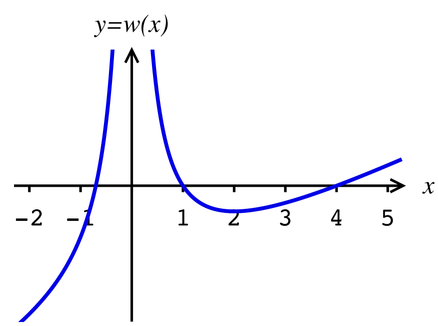
f. $\big((3x+1)(2x^2+4)\big)^{2/3}$

4(12 pts). Use **the definition of the derivative** to find $u'(x)$ if $u(x) = 3x^2 - 4$.
You can use the power rule to check your answer, but to receive credit on this problem, you must find the derivative using its definition as a limit.

5(6 pts). See the graph of $w(x)$.

a. On what interval(s) is $w(x)$ increasing?

b. On what interval(s) is $w(x)$ decreasing?



6. A coffee wholesaler who produces and sells x pounds of coffee finds that her revenue function is $R(x) = 50x - 0.2x^2$ and her cost function is $C(x) = 0.1x^2 + 8x + 150$.

a(5 pts). Find her profit function.

b(5 pts). Find her marginal profit function.

c(10 pts). She can produce between $x = 0$ and $x = 100$ pounds of coffee. What is her maximum possible profit? At what x will she see that profit?

7(19 pts). Let $t(x) = 16x + \dfrac{1}{x}$. Find the following.

a. $t'(x)$

b. An equation of the line tangent to the graph of $t(x)$ at the point $(1, 17)$.

c. The critical points of $t(x)$.

d. The x-interval(s) on which $t(x)$ is decreasing.

e. The x-value(s) at which $t(x)$ has a local maximum.

f. The x-value(s) at which $t(x)$ has a local minimum.

There were 5 extra credit points on this exam

1a(2 pts).(Source: 2.4) $f(x)$ is a rational function, so it is discontinous only where $3x-4=0$, or $x=\frac{4}{3}$.
Alternate form: $f(x)$ is discontinous only where $3x-2=0$, or $x=\frac{2}{3}$.

1b(2 pts).(Source: 2.4) A piecewise polynomial-defined function like $g(x)$ is continuous everywhere except possiblly its breakpoints. In this case $g(x)$ is discontinuous at its breakpoint $x=2$ because the limits there from the left and from the right disagree:

$$\lim_{x\to 2^-} g(x) = \lim_{x\to 2^-} 2x+1 = 5 \quad \text{but} \quad \lim_{x\to 2^+} g(x) = \lim_{x\to 2^+} 3-x = 1$$

That is, $g(x)$ has a jump discontinuity at $x=2$.
Alternate form:

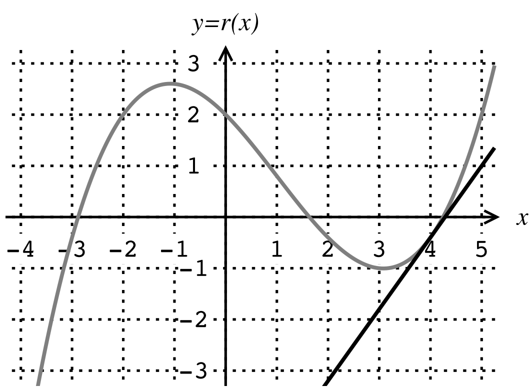
$$\lim_{x\to 1^-} g(x) = \lim_{x\to 1^-} 2x+1 = 3 \quad \text{but} \quad \lim_{x\to 1^+} g(x) = \lim_{x\to 1^+} 3-x = 2$$

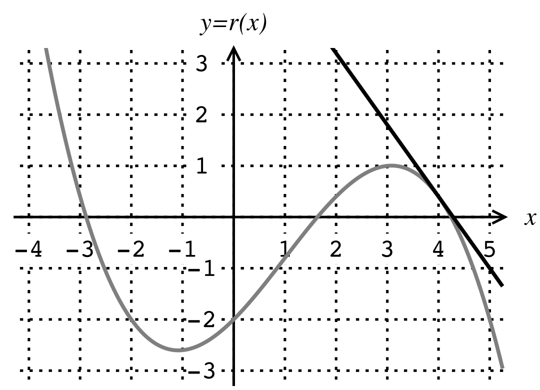
and so $\lim_{x\to 1} g(x)$ does not exist. Therefore, $g(x)$ is discontinuous at $x=1$.

2a(6 pts).(Source: 2.5) $\dfrac{\Delta r}{\Delta x} = \dfrac{r(3)-r(-2)}{3-(-2)} = \dfrac{-1-2}{3-(-2)} = -\dfrac{3}{5}$.
Alternate form: $\dfrac{\Delta r}{\Delta x} = \dfrac{r(3)-r(-2)}{3-(-2)} = \dfrac{1-(-2)}{3-(-2)} = \dfrac{3}{5}$.

2b(6 pts). The instantaneous rate of change of $r(x)$ at $x=4$ is the slope of the line tangent to the graph of $r(x)$ there. Sketch the line:

Alternate form:





In the graph on the left, the slope of that line appears to be about -1.5. (In fact, it is exactly -1.4.) On the right, the slope appears to be about 1.5. (In fact, it is exactly 1.4.)

3. a. (3 pts) $\left(\frac{1}{20}x^4 - 4x^{1/2} + 25\right)' = (\frac{1}{20})4x^3 - (4)\frac{1}{2}x^{-1/2} + 0$, or $= \frac{1}{5}x^3 - 2x^{-1/2}$.
Alternate form: $\left(\frac{1}{16}x^4 - 6x^{1/2} + 25\right)' = (\frac{1}{16})4x^3 - (6)\frac{1}{2}x^{-1/2} + 0 = \frac{1}{4}x^3 - 3x^{-1/2}$.

b.(4 pts) Chain Rule: derivative $= 5(8x^2-1)^4(8x^2-1)' = 5(8x^2-1)^4 16x$, or $80x(8x^2-1)^4$.
Alternate form: derivative $= 5(6x^2-1)^4(6x^2-1)' = 5(6x^2-1)^4 12x$, or $60x(6x^2-1)^4$.

c.(4 pts) If we let $b = (8x^2 - 1)^5$, then

$$\big(4x(8x^2-1)^5\big)' = (4x)'b + (4x)b' = 4(8x^2-1)^5 + 4x \cdot 80x(8x^2-1)^4.$$

Alternate form: Let $b = (6x^2 - 1)^5$. Then

$$\big(4x(6x^2-1)^5\big)' = (4x)'b + (4x)b' = 4(6x^2-1)^5 + 4x \cdot 60x(6x^2-1)^4.$$

d.(5 pts) Quotient Rule:

$$\left(\frac{4x-2}{3x^2+x}\right)' = \frac{(4x-2)'(3x^2+x) - (4x-2)(3x^2+x)'}{(3x^2+x)^2} = \frac{4(3x^2+x) - (4x-2)(6x+1)}{(3x^2+x)^2}$$

Alternate form:

$$\left(\frac{4x-2}{4x^2+x}\right)' = \frac{(4x-2)'(4x^2+x) - (4x-2)(4x^2+x)'}{(4x^2+x)^2} = \frac{4(4x^2+x) - (4x-2)(8x+1)}{(4x^2+x)^2}$$

e. (9 pts) Start with the quotient rule:

$$\left(\frac{5x+2}{\sqrt{7x+1}}\right)' = \frac{(5x+2)'\sqrt{7x+1} - (5x+2)\sqrt{7x+1}'}{(\sqrt{7x+1})^2}$$

The next step requires the quotient rule.

$$\frac{5(7x+1)^{1/2} - (5x+2)\frac{1}{2}(7x+1)^{-1/2}(7x+1)'}{(\sqrt{7x+1})^2} = \frac{5(7x+1)^{1/2} - (5x+2)\frac{1}{2}(7x+1)^{-1/2}7}{7x+1}$$

Alternate form:

$$\frac{7(5x+1)^{1/2} - (7x+2)\frac{1}{2}(5x+1)^{-1/2}(5x+1)'}{(\sqrt{5x+1})^2} = \frac{7(5x+1)^{1/2} - (7x+2)\frac{1}{2}(5x+1)^{-1/2}5}{5x+1}$$

f. (7 pts) First use the Chain Rule:

$$\Big(\big((2x+1)(3x^2+4)\big)^{2/3}\Big)' = \frac{2}{3}\big((2x+1)(3x^2+4)\big)^{-1/3}\big((2x+1)(3x^2+4)\big)'$$

Now use the product rule:

$$\begin{aligned} &= \frac{2}{3}\big((2x+1)(3x^2+4)\big)^{-1/3}\big((2x+1)'(3x^2+4) + (2x+1)(3x^2+4)'\big) \\ &= \frac{2}{3}\big((2x+1)(3x^2+4)\big)^{-1/3}\big(2(3x^2+4) + (2x+1)6x\big) \end{aligned}$$

Alternate form:

$$\begin{aligned}
&= \frac{2}{3}\big((3x+1)(2x^2+4)\big)^{-1/3}\big((3x+1)'(2x^2+4)+(3x+1)(2x^2+4)'\big) \\
&= \frac{2}{3}\big((3x+1)(2x^2+4)\big)^{-1/3}\big(3(2x^2+4)+(3x+1)4x\big)
\end{aligned}$$

4(12 pts).(Source: 2.7) The derivative is defined to be this limit:

$$\begin{aligned}
u'(x) &= \lim_{h\to 0}\frac{u(x+h)-u(x)}{h} = \lim_{h\to 0}\frac{3(x+h)^2-4-(3x^2-4)}{h} \\
&= \lim_{h\to 0}\frac{3(x^2+2xh+h^2)-4-(3x^2-4)}{h} \\
&= \lim_{h\to 0}\frac{3x^2+6xh+3h^2-4-3x^2+4}{h}
\end{aligned}$$

collect like terms in the numerator and then cancel the common factor h:

$$= \lim_{h\to 0}\frac{6xh+3h^2}{h} = \lim_{h\to 0}\frac{h(6x+3h)}{h} = \lim_{h\to 0}\frac{(6x+3h)}{1}$$

Now let $h \to 0$, and the deriative is $u'(x) = 6x$

Alternate form:

$$\begin{aligned}
u'(x) &= \lim_{h\to 0}\frac{u(x+h)-u(x)}{h} = \lim_{h\to 0}\frac{3(x+h)^2-4-(3x^2-4)}{h} \\
&= \lim_{h\to 0}\frac{3(x^2+2xh+h^2)-4-(3x^2-4)}{h} \\
&= \lim_{h\to 0}\frac{3x^2+6xh+3h^2-4-3x^2+4}{h}
\end{aligned}$$

collect like terms in the numerator and then cancel the common factor h:

$$= \lim_{h\to 0}\frac{6xh+3h^2}{h} = \lim_{h\to 0}\frac{h(6x+3h)}{h} = \lim_{h\to 0}\frac{(6x+3h)}{1}$$

Now let $h \to 0$, and the deriative is $u'(x) = 6x$

5(6 pts).(Source: 2.1)

a. $w(x)$ is increasing on $(-\infty,-2)$ and on $(0,\infty)$ Alternate form: $w(x)$ is increasing on $(-\infty,0)$ and on $(2,\infty)$

b. $w(x)$ is decreasing on $(-2,0)$. Alternate form: $w(x)$ is decreasing on $(0,2)$.

6a(5 pts).(Source: 2.9, 3.6) Profit is reventue minus cost: $P(x) = R(x) - C(x) = 50x - 0.2x^2 - (0.1x^2 + 8x + 180 = 42x - 0.3x^2 - 180$.

Alternate form: $P(x) = R(x) - C(x) = 50x - 0.2x^2 - (0.1x^2 + 8x + 150 = 42x - 0.3x^2 - 150$.

6b(5 pts). Marginal profit is $P'(x) = 42 - 0.6x$.

6c(10 pts). Solve $P'(x) = 42 - 0.6x = 0$ to find critical point is $x = 42 \div 0.6 = 70$.

To find absolute extrema, evaluate $P(x)$ at $x = 0$, 100, and 70:

$$P(0) = -180 \qquad P(70) = 1380 \qquad P(100) = 1080$$

Conclusion: Abs max profit is \$1380 at $x = 70$ pounds of coffee.
Alternate form:

$$P(0) = -150 \qquad P(70) = 1350 \qquad P(100) = 1050$$

Conclusion: Abs max profit is \$1350 at $x = 70$ pounds of coffee.
d(3 pts).(Source: 2.7,3.5) $t(x) = 25x + x^{-1}$, so $t';(x) = 25 - x^{-2}$

6e(4 pts). Slope equals $t'(1) = 24$. In point-slope form, the line is $y - 26 = 24(x - 1)$.

6f.(4 pts) Solve $0 = t'(x) = 25 - x^{-2}$

$$x^{-2} = 25 \implies x^2 = \frac{1}{25} \implies x = \pm\sqrt{\frac{1}{25}} = \pm\frac{1}{5}$$

6g(4 pts). Here's what we know so far about the signs of t':

$t'(x)$:	0	DNE	0
x :	$-\frac{1}{5}$	0	$\frac{1}{5}$

To complete the sign chart, evaluate $t'(x)$ at sample points chosen to the left of $-\frac{1}{5}$, between $-\frac{1}{5}$ and 0, between 0 and $\frac{1}{5}$, and to the right of $\frac{1}{5}$.

x	-10	$-.1$	$.1$	10
$t'(x)$	$25 - (-10)^{-2}$ 24.99 $+$	$25 - (-.1)^{-2}$ -75 $-$	$25 - .1^{-2}$ -75 $-$	$25 - 10^{-2}$ 24.99 $+$

Conclusion:

$t'(x)$:	$++++0------$	DNE	$------0++++$
x :	$-\frac{1}{5}$	0	$\frac{1}{5}$

$t(x)$ is decreasing where $t'(x) < 0$, that is, on $(-\frac{1}{5}, 0)$ and $(0, \frac{1}{5})$.
The signs of $t'(x)$ allow us to use the first derivative test:
6h(2 pts). $t(x)$ has a local maximum at $x = -\frac{1}{5}$.
6i(2 pts). $t(x)$ has a local minimum at $x = \frac{1}{5}$.

Alternate form:
6a(3 pts).(Source: 2.7,3.5) $t(x) = 16x + x^{-1}$, so $t';(x) = 16 - x^{-2}$

6b(4 pts). Slope equals $t'(1) = 15$. In point-slope form, the line is $y - 17 = 15(x - 1)$.

6c.(4 pts) Solve $0 = t'(x) = 16 - x^{-2}$

$$x^{-2} = 16 \quad \Longrightarrow \quad x^2 = \frac{1}{16} \quad \Longrightarrow \quad x = \pm\sqrt{\frac{1}{16}} = \pm\frac{1}{4}$$

6d(4 pts). Here's what we know so far about the signs of t':

$t'(x)$:	0	DNE	0
x :	$-\frac{1}{4}$	0	$\frac{1}{4}$

To complete the sign chart, evaluate $t'(x)$ at sample points chosen to the left of $-\frac{1}{4}$, between $-\frac{1}{4}$ and 0, between 0 and $\frac{1}{4}$, and to the right of $\frac{1}{4}$.

x	-10	$-.1$	$.1$	10
$t'(x)$	$16 - (-10)^{-2}$ 15.99 $+$	$16 - (-.1)^{-2}$ -84 $-$	$16 - .1^{-2}$ -84 $-$	$16 - 10^{-2}$ 15.99 $+$

Conclusion:

$t'(x)$:	$++++\,0\,------$	DNE	$------\,0\,++++$
x :	$-\frac{1}{4}$	0	$\frac{1}{4}$

$t(x)$ is decreasing where $t'(x) < 0$, that is, on $(-\frac{1}{4}, 0)$ and $(0, \frac{1}{4})$.

The signs of $t'(x)$ allow us to use the first derivative test:

6e(2 pts). $t(x)$ has a local maximum at $x = -\frac{1}{4}$.

6f(2 pts). $t(x)$ has a local minimum at $x = \frac{1}{4}$.