You may use an approved calculator, but no notes, books, phones, smart watches, earbuds, other electronic devices, or any other outside material. Notes stored in your calculator in the form of programs, equations, etc., are not allowed. You may not share a calculator with another student. Do not leave the room during the exam with a phone.
Read each problem carefully and simplify your answers.
Unless otherwise indicated, supporting work will be required on every problem worth more than 2 points.

1(12 pts). Find the domain of the given function.

a. $7x - 2$ b. $\dfrac{3}{\sqrt{x^2+7}}$ c. $\sqrt{x+7}$ d. $\dfrac{x}{x^2+6x+9}$

2(24 pts). Find all solutions x to each of the given equations.

a. $|x| = 13$ b. $4(x-2)+3 = (x-1)+(3x-5)$ c. $25x^2 - 16 = 0$

d. $25x^2 + 16x = 0$ e. $x^2 - 8x = 20$ f. $\dfrac{1}{5}x - 2 = \dfrac{1}{3}x$

3(4 pts). Find the slope of the given line, if it exists.

a. $2x - 5y = 1$ b. $y = 2$

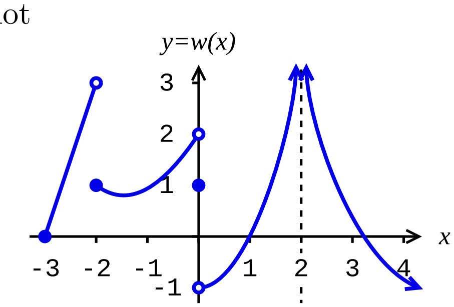
4(4 pts). Find an equation for the line with slope 8 passing through the point $(3, -4)$. Slope-intercept form is not required.

5(5 pts). Use the graph of $w(x)$ to find the following. Your answer to each should be a number, ∞, $-\infty$, or "does not exist."

a. $\lim\limits_{x\to -2} w(x)$ b. $\lim\limits_{x\to -2^-} w(x)$

c. $\lim\limits_{x\to 2} w(x)$ d. $\lim\limits_{x\to 0^+} w(x)$

e. $w(0)$



6(12 pts). Let $f(x) = x^2 + 4x$ and $g(x) = \frac{2}{x}$. Find the following and simplify your answers.

a. $f(x) \div g(x)$ b. $f(x)g(x)$ c. $f(g(x))$

7(10 pts). A bee keeper sells honey. To sell $x = 50$ jars of honey per month, she must set the price at \$16 per jar. For each \$2 reduction in the price, the bee keeper can expect to sell an additional 6 jars of honey per month. Find the price of honey $D(x)$ (dollars per jar) at which she can sell x jars per month, assuming $D(x)$ is a linear function of x.

8(5 pts). A syrup maker finds that he can sell x bottles of maple syrup per month at the price $D(x) = 40 - \frac{1}{2}x$ dollars per bottle. Express the syrup maker's revenue $R(x)$ as a function of x, the number of bottles he sells per month.

9(4 pts). What limit $\lim_{x \to ?} ? = ?$ is suggested by the table? Supporting work not required.

x	2.5	2.05	2.005	2.0005	$\cdots$
$q(x)$	5.7	5.97	5.997	5.9997	$\cdots$

10(20 pts). Find the given limit.

a. $\lim_{x \to -2} \dfrac{x+2}{x^2+9x+14}$

b. $\lim_{x \to -2^-} \dfrac{1+2x}{x+2}$

c. $\lim_{x \to \infty} \dfrac{4x^2+1}{7x^2-2x+101}$

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1(12 pts). Find the domain of the given function.

a. $7x - 3$ b. $\dfrac{3}{\sqrt{x+5}}$ c. $\sqrt{x^2+5}$ d. $\dfrac{x}{x^2-4x+4}$

2(24 pts). Find all solutions x to each of the given equations.

a. $|x| = -12$ b. $4(x-2)+3 = (x-1)+(3x-4)$ c. $16x^2 - 25 = 0$

d. $16x^2 + 25x = 0$ e. $x^2 - 8x = -15$ f. $\dfrac{1}{5}x - 3 = \dfrac{1}{2}x$

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a. $2x - 3y = 1$ b. $x = 2$

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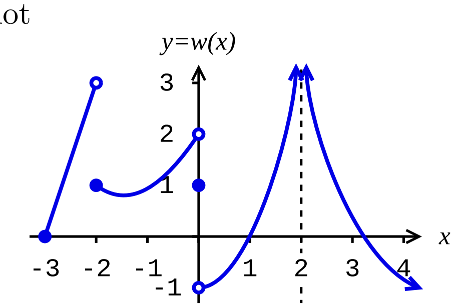
e. $w(0)$



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a. $\displaystyle\lim_{x\to -2} \frac{x+2}{x^2+7x+10}$ b. $\displaystyle\lim_{x\to -2^+} \frac{1+2x}{x+2}$ c. $\displaystyle\lim_{x\to \infty} \frac{5x^2+1}{7x^2-2x+101}$

1(12 pts).(Source: 1.1) See the domain rules in the review notes, section 1.1, found here: https://kunklet.people.charleston.edu/MATH116/Z116review.pdf

1a. The linear function $7x - 3$ is defined for all real numbers. That is, its domain is $(-\infty, \infty)$.

1b and c. Any function of the form $\sqrt{\heartsuit}$ is defined only when $\heartsuit \geq 0$. Furthermore, because division by zero is impossible, any function of the form $\frac{3}{\sqrt{\heartsuit}}$ is defined only when $\heartsuit > 0$.

1b. $x^2 + 7 \geq 7 > 0$ (because $x^2 \geq 0$) for all real numbers x. The domain of $\frac{3}{\sqrt{x^2+7}}$ is $(-\infty, \infty)$.

Alternate form: $\frac{3}{\sqrt{x+5}}$ is defined exactly when $x + 5 > 0$, or $x > -5$. The domain is $(-5, \infty)$.

1c. $\sqrt{x+7}$ is defined when $x + 7 \geq 0$, or $x \geq -7$. The domain is $[-7, \infty)$.

Alternate form: $x^2 + 5 > 0$ (because $x^2 \geq 0$) for all real numbers, so $\sqrt{x^2+5}$ is defined on the entire real number line: $(-\infty, \infty)$.

1d. Factor the denominator: $\frac{x}{x^2+6x+9} = \frac{x}{(x+3)^2}$ is defined for all x other than -3, where the denominator is zero. In interval form, this is $(-\infty, -3) \cup (-3, \infty)$.

Alternate form: The domain of $\frac{x}{x^2-4x+4} = \frac{x}{(x-2)^2}$ is all x other than 2, or $(-\infty, 2) \cup (2, \infty)$.

2a(2 pts).(Source: 0.1) The only numbers x for which $|x| = 13$ are $x = 13$ and $x = -13$.

Alternate form: $|x| \geq 0$ for all x, so $|x| = -12$ has no solutions.

2b.(3 pts).(Source: 0.6)

$$\begin{aligned}
4(x-2) + 3 &= (x-1) + (3x-5) \\
4x - 8 + 3 &= x - 1 + 3x - 5 \\
4x - 8 + 3 &= 4x - 6 \\
4x - 5 &= 4x - 6 \\
0 &= -1
\end{aligned}$$

There's no x that will make this last equation true, so the equation has **no solutions**.

Alternate form:

$$\begin{aligned}
4(x-2) + 3 &= (x-1) + (3x-4) \\
4x - 8 + 3 &= x - 1 + 3x - 4 \\
4x - 8 + 3 &= 4x - 5 \\
4x - 5 &= 4x - 5 \\
0 &= 0
\end{aligned}$$

This is true regardless of the value of x. The solution set is **all real numbers**.

2c(4 pts).(Source: 0.7)

$$25x^2 - 16 = 0 \quad \Longrightarrow \quad 25x^2 = 16 \quad \Longrightarrow \quad x^2 = \frac{16}{25} \quad \Longrightarrow \quad x = \pm\sqrt{\frac{16}{25}} = \pm\frac{4}{5}$$

Alternate form:

$$16x^2 - 25 = 0 \quad \Longrightarrow \quad 16x^2 = 25 \quad \Longrightarrow \quad x^2 = \frac{25}{16} \quad \Longrightarrow \quad x = \pm\sqrt{\frac{25}{16}} = \pm\frac{5}{4}$$

2d(5 pts).(Source: 0.7) Factor the left side:

$$x(25x+16)=0$$
$$x=0 \quad\text{or}\quad 25x+16=0$$
$$x=0 \quad\text{or}\quad x=-\frac{16}{25}$$

Alternate form:

$$x(16x+25)=0$$
$$x=0 \quad\text{or}\quad 16x+25=0$$
$$x=0 \quad\text{or}\quad x=-\frac{25}{16}$$

2e(5 pts).(Source: 0.7) Get a zero on one side and factor the other.

$$x^2-8x-20=0 \quad\Longrightarrow\quad (x+2)(x-10)=0 \quad\Longrightarrow\quad x=-2 \text{ or } x=10$$

Alternate form:

$$x^2-8x+15=0 \quad\Longrightarrow\quad (x-3)(x-5)=0 \quad\Longrightarrow\quad x=3 \text{ or } x=5$$

2f(5 pts).(Source: 0.6)

$$\frac{1}{5}x-2=\frac{1}{3}x \quad\Longrightarrow\quad \frac{1}{5}x-\frac{1}{3}x=2 \quad\Longrightarrow\quad \left(\frac{1}{5}-\frac{1}{3}\right)x=2$$

Get a common denominator and add the fractions

$$\left(\frac{3}{15}-\frac{5}{15}\right)x=2 \quad\Longrightarrow\quad -\frac{2}{15}x=2 \quad\Longrightarrow\quad x=2\left(-\frac{15}{2}\right)=-15$$

Alternate form:

$$\frac{1}{5}x-2=\frac{1}{3}x \quad\Longrightarrow\quad \frac{1}{5}x-\frac{1}{3}x=2 \quad\Longrightarrow\quad \left(\frac{1}{5}-\frac{1}{3}\right)x=2$$

$$\Longrightarrow\quad \left(\frac{3}{15}-\frac{5}{15}\right)x=2 \quad\Longrightarrow\quad -\frac{2}{15}x=2 \quad\Longrightarrow\quad x=2\left(-\frac{15}{2}\right)=-15$$

3a(4 pts).(Source: 0.5) Write the line's equation in slope-intercept form:

$$2x-5y=1 \qquad -5y=-2x+1 \qquad y=\frac{2}{5}x-\frac{1}{5}$$

The slope is the coefficient of x, that is, $\frac{2}{5}$.

Alternate form:

$$2x-3y=1 \qquad -3y=-2x+1 \qquad y=\frac{2}{3}x-\frac{1}{3}$$

The slope is $\frac{2}{3}$.

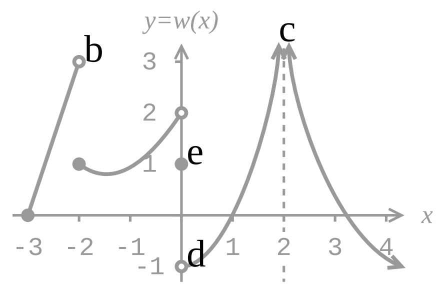
3b(0 pts).(Source: 0.5) The line $y = 2$ is horizontal. Its slope is 0.
Alternate form: $x = 2$ is vertical. It has **no slope**.

4(4 pts).(Source: 0.5) The line can be expressed in point-slope form as $y + 4 = 8(x - 3)$
Alternate form: $y + 3 = 7(x + 2)$.

5(5 pts).(Source: 2.1) a. $\lim_{x\to-2} w(x)$ does not exist because $\lim_{x\to-2^-} w(x) \neq \lim_{x\to-2^+} w(x)$.

b. $\lim_{x\to-2^-} w(x) = 3$ c. $\lim_{x\to 2} w(x) = +\infty$

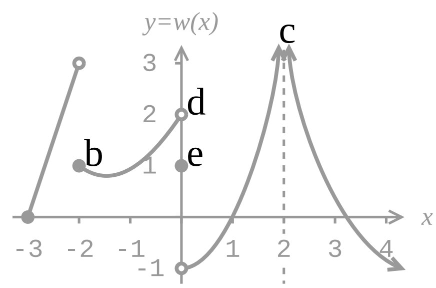
d. $\lim_{x\to 0^+} w(x) = -1$ e. $w(0) = 1$



Alternate form: a. $\lim_{x\to-2} w(x)$ does not exist because $\lim_{x\to-2^-} w(x) \neq \lim_{x\to-2^+} w(x)$.

b. $\lim_{x\to-2^+} w(x) = 1$ c. $\lim_{x\to 2} w(x) = +\infty$

d. $\lim_{x\to 0^-} w(x) = 2$ e. $w(0) = 1$



6a(12 pts).(Source: 1.2) $f(x) \div g(x) = (x^2 + 4x) \div \frac{2}{x} = (x^2 + 4x) \cdot \frac{x}{2} = \frac{x^3}{2} + 2x^2$
Alternate form: $f(x) \div g(x) = (x^2 + 5x) \div \frac{2}{x} = (x^2 + 5x) \cdot \frac{x}{2} = \frac{x^3}{2} + \frac{5}{2}x^2$

6b(0 pts). $f(x)g(x) = (x^2 + 4x) \cdot \frac{2}{x} = x^2 \cdot \frac{2}{x} + 4x \cdot \frac{2}{x} = 2x + 8$
Alternate form: $f(x)g(x) = (x^2 + 5x) \cdot \frac{2}{x} = x^2 \cdot \frac{2}{x} + 5x \cdot \frac{2}{x} = 2x + 10$

6c(0 pts). $f(\quad) = (\quad)^2 + 4(\quad)$, so $f\big(g(x)\big) = f\left(\frac{2}{x}\right) = \left(\frac{2}{x}\right)^2 + 4\left(\frac{2}{x}\right) = \frac{4}{x^2} + \frac{8}{x}$.
Alternate form: $f\big(g(x)\big) = f\left(\frac{2}{x}\right) = \left(\frac{2}{x}\right)^2 + 5\left(\frac{2}{x}\right) = \frac{4}{x^2} + \frac{10}{x}$.

7(10 pts).(Source: 1.3) To say that $D(x)$ is a linear function of x means that the graph of $D(x)$ is a line. According to the third sentence, the slope of the line is $\Delta\text{price} \div \Delta\text{quantity} = \frac{-2}{+6} = -\frac{1}{3}$. The line can be written in the point-slope form $D - 16 = -\frac{1}{3}(x - 50)$, so $D(x) = 16 - \frac{1}{3}(x - 50)$, or $-\frac{1}{3}x + 16 + \frac{50}{3} = -\frac{1}{3}x + \frac{48}{3} + \frac{50}{3} = -\frac{1}{3}x + \frac{98}{3}$.
Alternate form: Slope of the line is $-\frac{1}{3}$ as above. The line can be written in the point-slope form $D - 18 = -\frac{1}{3}(x - 50)$, so $D(x) = 18 - \frac{1}{3}(x - 50)$, or $-\frac{1}{3}x + 18 + \frac{50}{3} = -\frac{1}{3}x + \frac{54}{3} + \frac{50}{3} = -\frac{1}{3}x + \frac{104}{3}$.

8(5 pts).(Source: 1.3) Revenue equals price times quantity: $R(x) = D(x)x = (40 - \frac{1}{2}x)x$, or $40x - \frac{1}{2}x^2$

9(4 pts).(Source: 2.1) $\lim_{x\to 2^+} q(x) = 6$

10a(1 pts).(Source: 2.2,2.3) . This limit is in the indeterminate form "$\frac{0}{0}$." Find and cancel the common factor:

$$\lim_{x\to -2} \frac{x+2}{x^2+9x+14} = \lim_{x\to -2} \frac{x+2}{(x+2)(x+7)} = \lim_{x\to -2} \frac{1}{x+7} = \frac{1}{5}$$

Alternate form:

$$\lim_{x\to -2} \frac{x+2}{x^2+7x+10} = \lim_{x\to -2} \frac{x+2}{(x+2)(x+5)} = \lim_{x\to -2} \frac{1}{x+5} = \frac{1}{3}$$

10b(0 pts).(Source: 2.2,2.3) $\lim_{x\to -2^-} \frac{1+2x}{x+2} =$ "$\frac{-3}{0}$" which indicates an infinite limit. Sign analysis:

$$x \to -2^- \quad \Longrightarrow \quad x < -2 \quad \Longrightarrow \quad x+2 < 0 \quad \Longrightarrow \quad \frac{1+2x}{x+2} = \frac{-}{-} = +,$$

so the infinite limit must be $+\infty$.
Alternate form:

$$x \to -2^+ \quad \Longrightarrow \quad x > -2 \quad \Longrightarrow \quad x+2 > 0 \quad \Longrightarrow \quad \frac{1+2x}{x+2} = \frac{-}{+} = -,$$

so the infinite limit must be $-\infty$.

10c(0 pts).(Source: 2.3) See **Fact** 2.3.r11 and example 2.3.r12 in the review notes.

$$\lim_{x\to\infty} \frac{4x^2+1}{7x^2-2x+101} = \lim_{x\to\infty} \frac{4x^2}{7x^2} = \lim_{x\to\infty} \frac{4}{7} = \frac{4}{7}$$

Alternate form:

$$\lim_{x\to\infty} \frac{5x^2+1}{7x^2-2x+101} = \lim_{x\to\infty} \frac{5x^2}{7x^2} = \lim_{x\to\infty} \frac{5}{7} = \frac{5}{7}$$