

More problems for section 5.4 of *Essentials of Precalculus with Calculus Previews* by Zill and Dewar, 6e.

1. When I opened my savings account, I deposited \$200, but since then I haven't made any deposits or withdrawals. After 7 years, my money has grown with interest to \$260.
  - a. Express the balance in my account as a function of  $t$ , where  $t$  is the number of years since I opened the account.
  - b. Find the balance in my account after 3.5 years.
  - c. How long will it take for my original deposit to grow to \$300?
2. A bacterial culture initially contains 100 cells and grows exponentially. After one hour, the population has increased to 420 cells.
  - a. Express the number of bacteria in the population as a function of  $t$ , where  $t$  is the number of hours since time 0.
  - b. Find the number of bacteria after 3 hours.
  - c. When will the population reach 10,000?
3. The table below gives estimates of world population (measured in millions of people) from 1750 to 2000:

| Year | Population | Year | Population | Year | Population |
|------|------------|------|------------|------|------------|
| 1750 | 790        | 1850 | 1260       | 1950 | 2560       |
| 1800 | 980        | 1900 | 1650       | 2000 | 6080       |

- a. Assuming exponential growth, use the population figures for 1750 and 1800 to predict world population in 1900 and 1950. Compare with the actual figures.
- b. Assuming exponential growth, use the population figures for 1850 and 1900 to predict world population in 1950. Compare with the actual figure.
- c. Assuming exponential growth, use the population figures for 1900 and 1950 to predict world population in 2000. Compare with the actual figure.
- d. What can account for the differences between your predictions and the actual populations?
4. Bismuth-210 has a half-life of 5 days.
  - a. A sample of Bismuth-210 originally has a mass of 800 mg. Find a formula for the mass remaining after  $t$  days.
  - b. Find the mass after 13 days.
  - c. When will the mass be reduced to 1mg?
5. A sample of tritium-3 decayed to 94.5% of its original mass in 1 year.
  - a. Find the half-life of tritium-3.
  - b. How long would it take for a sample to decay to 20% of its original mass?

#### Answers

- 1a.  $\$200e^{(t \ln 1.3)/7}$ , or  $\$200(1.3)^{t/7}$     1b.  $\$200(1.3)^{1/2}$ , or approximately \$228.04    1c.  $(7 \ln 1.5)/(\ln 1.3)$  (exact answer), or 10.8180 years (approximate answer)    2a.  $100(4.2)^t$     2b. approximately 7409    2c. At  $t = (\ln 100)/(\ln 4.2)$  (exact answer), or 3.209 hours (approximate answer)    3a. 1508.08 million and 1870.78 million    3b. 2160.71 million    3c. 3971.88 million
- 3d. The exponential growth model assumes a constant net growth rate = birth rate - death rate. Over history, the birth and death rates can change depending on wars, epidemics, availability of food, advances in medicine, etc..    4a.  $800(0.5)^{t/5}$
- 4b.  $800(0.5)^{13/5}$ , or approximately 131.95 mg    4c. At  $t = (5 \ln 800)/(\ln 2)$  (exact answer), or 48.22 days (approximate answer)
- 5a.  $(\ln 0.5)/(\ln 0.945)$  years    5b.  $(\ln 0.2)/(\ln 0.945)$  (approximately 28.45) years