

More problems for section 2.6 of *Essentials of Precalculus with Calculus Previews* by Zill and Dewar, 6e.

The **intersection** of two sets consists of all points the two sets have in common. The domain of $f + - \times g$ is the intersection of the domain of f and the domain of g (even if $f + - \times g$ simplifies to a function with a larger domain). The domain of $f \div g$ is the intersection of the domain of f and the domain of g , excluding any points where $g = 0$ (even if ...).

1. Find the domain for $f + g$, $f - g$, $f \times g$, and $f \div g$.

- a. $f(x) = \frac{1}{x-1}$, $g(x) = \frac{2}{x^2-2x+3}$
- b. $f(x) = \frac{1}{x^2-1}$, $g(x) = \frac{x}{x+1}$
- c. $f(x) = \frac{1}{x^2-9}$, $g(x) = \sqrt{4-x}$
- d. $f(x) = \frac{1}{x^2-4}$, $g(x) = \sqrt{1+x}$
- e. $f(x) = \frac{1}{x^2-4x+12}$, $g(x) = \sqrt{x-8}$
- f. $f(x) = \frac{1}{\sqrt{x-2}}$, $g(x) = \frac{x-3}{\sqrt{1+x}}$
- g. $f(x) = \sqrt{2-x}$, $g(x) = \sqrt{4+x}$
- h. $f(x) = \sqrt{x+1}$, $g(x) = \sqrt{8+2x-x^2}$
- i. $f(x) = \sqrt{-3-x}$, $g(x) = \sqrt{-x^2+2x+8}$
- j. $f(x) = \frac{1}{x+10}$, $g(x) = \sqrt{27-6x-x^2}$

2. Find $(f \circ g)(x)$ and $(g \circ f)(x)$ and the domain of each.

- a. $f(x) = \sqrt{x+2}$, $g(x) = \sqrt{2-x}$
- b. $f(x) = \sqrt{3-x}$, $g(x) = \sqrt{1-x}$
- c. $f(x) = \sqrt{3+x}$, $g(x) = \sqrt{2+x}$
- d. $f(x) = \sqrt{1+x}$, $g(x) = x^2-4$
- e. $f(x) = \sqrt{1-x}$, $g(x) = 2x^2+3$
- f. $f(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$, $g(x) = \begin{cases} 0 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$
- g. $f(x) = \begin{cases} -1 & \text{if } x > 0 \\ 1 & \text{if } x \leq 0 \end{cases}$, $g(x) = \begin{cases} 0 & \text{if } x > 0 \\ 1 & \text{if } x \leq 0 \end{cases}$
- h. $f(x) = \begin{cases} x & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$, $g(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ |x| & \text{if } x < 0 \end{cases}$
- i. $f(x) = \frac{1}{x+1}$, $g(x) = \frac{1}{x-1}$
- j. $f(x) = \frac{1}{x}$, $g(x) = \frac{x+2}{x}$
- k. $f(x) = \frac{x}{x+1}$, $g(x) = \frac{x}{x-1}$
- l. $f(x) = x^2+2x$, $g(x) = \sqrt{x-3}$
- m. $f(x) = x^2+2x$, $g(x) = \sqrt{3-x}$

Answers

1a. $f + - \times g : \mathbb{R} \setminus \{1, -1, 3\}$. $f \div g$: same. 1b. $f + - \times g : \mathbb{R} \setminus \{1, -1\}$. $f \div g : \mathbb{R} \setminus \{1, 0, -1\}$. 1c. $f + - \times g : (-\infty, -3) \cup (-3, 3) \cup (3, 4]$. $f \div g : (-\infty, -3) \cup (-3, 3) \cup (3, 4)$ 1d. $\pm \times : [-1, 2) \cup (2, \infty)$, $\div : (-1, 2) \cup (2, \infty)$ 1e. $\pm \times : [8, \infty)$, $\div : (8, \infty)$ 1f. $\pm \times : (2, \infty)$, $\div : (2, 3) \cup (3, \infty)$ 1g. $\pm \times : [-4, 2]$, $\div : (-4, 2]$ 1h. $\pm \times : [-1, 4]$, $\div : [-1, 4)$ 1i. $\pm \times : (-\infty, -3] \cup [4, \infty)$, $\div : (-\infty, -3] \cup (4, \infty)$ 1j. $\pm \times : [-9, 3]$, $\div : (-9, 3)$ 2a. $f \circ g = \sqrt{2 + \sqrt{2-x}}$, dom = $(\infty, 2]$; $g \circ f = \sqrt{2 - \sqrt{x+2}}$, dom = $[-2, 2]$. 2b. $f \circ g = \sqrt{3 - \sqrt{1-x}}$, dom = $[-8, 1]$; $g \circ f = \sqrt{1 - \sqrt{3-x}}$, dom = $[2, 3]$. 2c. $f \circ g = \sqrt{3 + \sqrt{2+x}}$, dom = $[-2, \infty)$; $g \circ f = \sqrt{2 + \sqrt{3+x}}$, dom = $[-3, \infty)$. 2d. $f \circ g = \sqrt{x^2-3}$, dom = $(-\infty, \sqrt{3}] \cup [\sqrt{3}, \infty)$; $g \circ f = x-3$, dom = $[-1, \infty)$. 2e. $f \circ g = \sqrt{-2-2x^2}$, dom = $\emptyset$; $g \circ f = 5-2x$, dom = $(-\infty, 1]$. 2f. $f \circ g = f$, dom = $\mathbb{R}$; $g \circ f = 0$, dom = $\mathbb{R}$. 2g. $f \circ g = -f$, dom = $\mathbb{R}$; $g \circ f = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases}$, dom = $\mathbb{R}$. 2h. $f \circ g = g$, dom = $\mathbb{R}$; $g \circ f = 1$, dom = $\mathbb{R}$. 2i. $f \circ g = \frac{x-1}{x}$, dom = $\mathbb{R} \setminus \{0, 1\}$; $g \circ f = -\frac{x+1}{x}$, dom = $\mathbb{R} \setminus \{-1, 0\}$. 2j. $f \circ g = \frac{x}{x+2}$, dom = $\mathbb{R} \setminus \{0, -2\}$; $g \circ f = 1+2x$, dom = $\mathbb{R} \setminus \{0\}$. 2k. $f \circ g = \frac{x}{2x-1}$, dom = $\mathbb{R} \setminus \{\frac{1}{2}, 1\}$; $g \circ f = -x$, dom = $\mathbb{R} \setminus \{-1\}$. 2l. $f \circ g = x-3+2\sqrt{x-3}$, dom = $[3, \infty)$; $g \circ f = \sqrt{x^2+2x-3}$, dom = $(\infty, -3] \cup [1, \infty)$. 2m. $f \circ g = 3-x+2\sqrt{3-x}$, dom = $(-\infty, 3]$; $g \circ f = \sqrt{3-2x-x^2}$, dom = $[-3, 1]$.