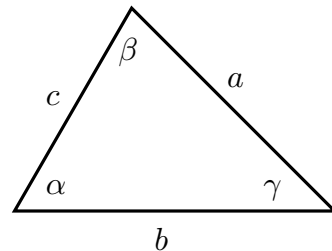


1 (10 pts). These 3 problems refer to the figure at the right.

- a. Find b if $a = 2$, $\beta = 110^\circ$ and $\gamma = 20^\circ$.
- b. Find c if $a = 1$, $b = 2$ and $\gamma = 10^\circ$.
- c. Find α if $a = 5$, $b = 6$ and $c = 2$.



Solution:

1a.(Source: 4.12.more.1d) This problem is classified as AAS (given two angles and one side). First find $\alpha = 180^\circ - \beta - \gamma = 50^\circ$. Then, by the Law of Sines,

$$\begin{aligned}\frac{b}{\sin \beta} &= \frac{a}{\sin \alpha} \\ \frac{b}{\sin 110^\circ} &= \frac{2}{\sin 50^\circ} \\ b &= \frac{2 \sin 110^\circ}{\sin 50^\circ}.\end{aligned}$$

1b.(Source: 4.12.more.1) This problem is classified as SAS (given two sides and the angle between them). By the Law of Cosines,

$$\begin{aligned}c^2 &= a^2 + b^2 - 2ab \cos \gamma \\ &= 1 + 4 - 4 \cos 10^\circ \\ c &= \sqrt{5 - 4 \cos 10^\circ}\end{aligned}$$

1c.(Source: 4.13.more.2) This problem is classified as SSS (given three sides). To find α , first find its cosine. By the Law of Cosines,

$$\begin{aligned}a^2 &= b^2 + c^2 - 2bc \cos \alpha \\ 25 &= 36 + 4 - 24 \cos \alpha \\ \cos \alpha &= \frac{15}{24} = \frac{5}{8}.\end{aligned}$$

As an angle inside a triangle, α must lie in $[0, \pi]$, which is exactly the range of the inverse cosine. Therefore, $\alpha = \cos^{-1}(\frac{5}{8})$.