

1 (10 pts). Sketch one cycle of the graph of $y = 1 - 2 \cos\left(\frac{\pi}{2}x + \frac{\pi}{4}\right)$. Your drawing should include one cycle of the curve. Draw the axes where you wish and label hashmarks so as to clearly indicate the coordinates of every point in your cycle where the cosine equals 0, 1, or -1 .

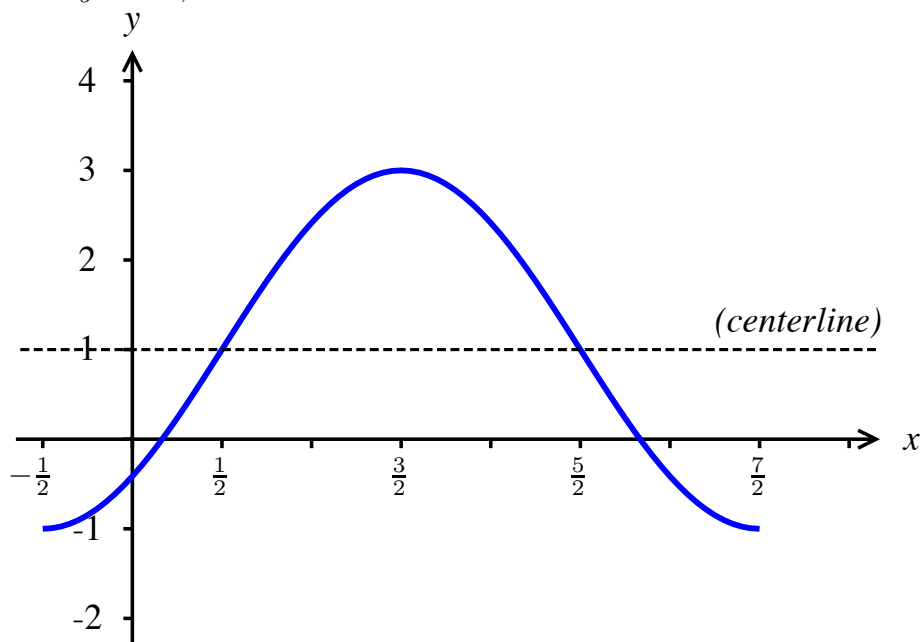
What is the y -intercept of this graph?

1. (Source: 4.3.more.1.gp) The function $1 - 2 \cos\left(\frac{\pi}{2}x + \frac{\pi}{4}\right)$ will go through one cycle when the angle inside the cosine goes from 0 to 2π .

$$\begin{aligned} 0 &\leq \frac{\pi}{2}x + \frac{\pi}{4} \leq 2\pi & -\frac{2}{\pi} \cdot \frac{\pi}{4} \leq x \leq \frac{2}{\pi} \cdot \frac{7\pi}{4} \\ -\frac{\pi}{4} &\leq \frac{\pi}{2}x \leq 2\pi - \frac{\pi}{4} = \frac{7\pi}{4} & -\frac{1}{2} \leq x \leq \frac{7}{2} \end{aligned}$$

So, our cycle of the cosine will start at $x = -\frac{1}{2}$ and end at $x = \frac{7}{2}$.

The coefficient -2 reflects the cosine across the x -axis and stretches it by a factor of 2. The 1 shifts the curve up one unit, so that the new centerline of the curve is $y = 1$. The minimum value of y is -1 , and the maximum value is 3.



The y intercept of the curve is $1 - 2 \cos\left(\frac{\pi}{2} \cdot 0 + \frac{\pi}{4}\right) = 1 - 2 \cos\left(\frac{\pi}{4}\right) = 1 - 2 \cdot \frac{\sqrt{2}}{2} = 1 - \sqrt{2}$. In particular, the y -intercept is negative, so your curve should intersect the y -axis below the origin.