No notes, books, electronic devices, or outside materials of any kind.

Read each problem carefully and simplify your answers.

Unless otherwise indicated, supporting work will be required on every problem worth more than 2 points.

1(6 pts). Find equations of all horizontal or slant asymptotes to $y = \dfrac{3x^2 - 3x + 1}{x + 5}$.

2(12 pts). A 6-ft-tall man is walking away from a lamp atop a 30-ft-tall lamppost. Express the length of the man's shadow cast on the ground by the lamp as a function of his distance from the lamppost. Assume that the ground is horizontal and that the man and lamppost are vertical.

3(8 pts). Find $f^{-1}(x)$ if $f(x) = \sqrt[3]{\frac{1}{x^3-8}}$.

4(14 pts). Find the difference quotient $\frac{f(x+h)-f(x)}{h}$ for $f(x) = x^3 - 2x^2 + 4$ and simplify the result to cancel the factor h from top and bottom.

5(9 pts). Sketch the graph of $y = -2(x+1)^3(x-1)^2$. Your graph needn't be to accurate scale but should show all intercepts, the graph's behavior at each, and the correct end behavior of the function.

6(16 pts). Sketch the graph of $w(x) = \frac{x^2+x-6}{x^2+5x+6}$.
Find and report all intercepts, asymptotes, and holes, or state that they do not exist. Give asymptotes by their equations; give holes and intercepts by their coordinates. You are not required to find intersections of the graph with any of its asymptotes.

7a(5 pts). List all possible rational zeros of $2x^4 + ax^3 + bx^2 + cx - 16$ if a, b, and c are integers.

7b(13 pts). Find all zeros and factor completely: $2x^4 - 15x^3 + 23x^2 + 24x - 16$.

8(5 pts). Find the value of k if $p(x) = x^3 - kx + 3$ is divisible by $x + 2$.

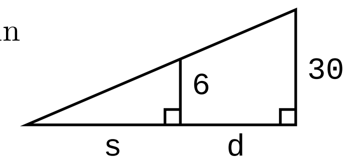
9(12 pts). Find all zeros and factor completely: $4x^3 + 5x^2 + 2x - 6$. Hint: $x = -1 + i$ is a zero.

1.(Source: 3.5.32-34) Since the degree of the numerator is exactly one more than that of the denominator, this graph has a slant asymptote. We can find it by long division, or, since the divisor is of the form $x - c$, by synthetic division. We can stop as soon as we find the quotient.

$$\begin{array}{r|rrr} -5 & 3 & -3 & 11 \\ & & -15 & \text{irrelevant} \\ \hline & 3 & -18 & \text{irrelevant} \end{array}$$

The slant asymptote is $y = 3x - 18$.

2.(Source: 2.9.21) See the figure. Our goal is to write s entirely in terms of d. By similar triangles,



$$\frac{s}{6} = \frac{s+d}{30} \qquad 24s = 6d$$
$$30s = 6s + 6d \qquad s = \frac{1}{4}d.$$

3.(Source: 2.8.more.1qr) Solve for x in the equation $y = f(x)$:

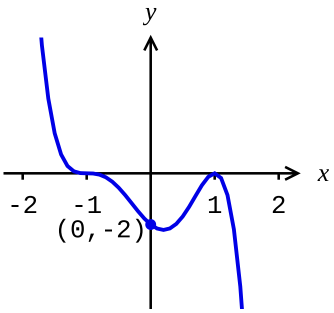
$$y = \sqrt[3]{\frac{1}{x^3 - 8}} \qquad \frac{1}{y^3} = x^3 - 8 \qquad x = \sqrt[3]{\frac{1}{y^3} + 8} = f^{-1}(y)$$
$$y^3 = \frac{1}{x^3 - 8} \qquad x^3 = \frac{1}{y^3} + 8 \qquad f^{-1}(x) = \sqrt[3]{\frac{1}{x^3} + 8}.$$

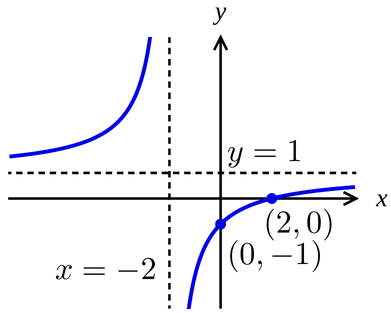
Note that $\sqrt[3]{a^3 - b^3} \neq a - b$, because $(a-b)^3 \neq a^3 - b^3$.

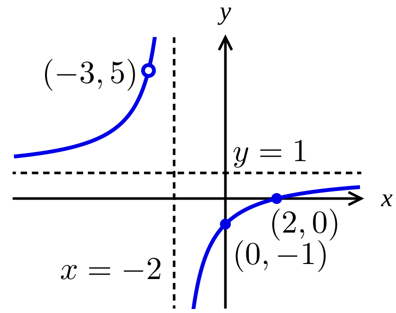
4.(Source: 2.10.17,18) $\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^3 - 2(x+h)^2 + 4 - (x^3 - 2x^2 + 4)}{h}$. Using the third row of Pascal's triangle 1 3 3 1, expand this to

$$\begin{aligned}
&\left(x^3 + 3x^2h + 3xh^2 + h^3 - 2(x^2 + 2xh + h^2) + 4 - (x^3 - 2x^2 + 4)\right) \cdot \frac{1}{h} \\
&= \left(x^3 + 3x^2h + 3xh^2 + h^3 - 2x^2 - 4xh - 2h^2 + 4 - x^3 + 2x^2 - 4\right) \cdot \frac{1}{h} \\
&= \left(3x^2h + 3xh^2 + h^3 - 4xh - 2h^2\right) \cdot \frac{1}{h} \\
&= \left(3x^2 + 3xh + h^2 - 4x - 2h\right) \cdot \frac{h}{h} \\
&= \left(3x^2 + 3xh + h^2 - 4x - 2h\right) \cdot \frac{h}{h} = 3x^2 + 3xh + h^2 - 4x - 2h.
\end{aligned}$$

5.(Source: 3.1.more.1b, 3.1,13-18) $-2(x+1)^3(x-1)^2 = -2x^5 +$ lower order terms, so the end behavior of this polynomial is $y \to \mp\infty$ as $x \to \pm\infty$. The zeros (and their multiplicities) are -1 (multiplicity 3) and 1 (multiplicity 2). The polynomial changes signs at its zero of odd multiplicity, -1. Its graph is tangent to the x-axis at both zeros, since these have multiplicity greater than 1. Plug in $x = 0$ to calculate the y-intercept: $-2(0+1)^3(0-1)^2 = -2$. See the graph below on the left.







5. $y = -2(x+1)^3(x-1)^2$ 6. $y = \frac{x-2}{x+2}$ 6. $y = \frac{(x+3)(x-2)}{(x+3)(x+2)}$

6.(Source: 3.5.more.1v-x) First factor the numerator and denominator: $w(x) = \frac{(x+3)(x-2)}{(x+3)(x+2)}$. Note that the common factor causes a hole at $x = -3$. Now cancel. We'll find the graph of $w(x)$ is the as the graph of $\frac{x-2}{x+2}$ except for this hole.

The x-intercept is the zero of the numerator, or 2. Let $x = 0$ and solve for the y-intercept $= \frac{-2}{2} = -1$. There's a vertical asymptote at the zero of the denominator, $x = -2$. When x is very large, $\frac{x-2}{x+2} \approx \frac{x}{x} = 1$, so $y = 1$ is a horizontal asymptote. The curve has no slant asymptote.

Here's a sign chart to help us graph:

$$\begin{array}{rl}
x-2: & ----------0+++++ \\
x+2: & ----0+++++++++++ \\
\dfrac{x-2}{x+2}: & ++++0----0++++ \\
\hline
x: & \qquad\quad -2 \qquad\qquad\quad 2
\end{array}$$

The graph appears above the x-axis when $y > 0$ and below the x-axis when $y < 0$. See the center graph above.

To graph the function in question, remove a hole at $x = -3$. The y-coordinate of this hole is $\frac{-3-2}{-3+2} = 5$. See the graph on the right above

7a.(Source: 3.3.39) The numerator of a rational zero must divide the constant term, 16, and the denominator must divide the lead coefficient 2. Since $\frac{2}{2} = 1$, $\frac{4}{2} = 2$, $\frac{8}{2} = 4$ and $\frac{16}{2} = 8$, the possible rational zeros are

$$1, -1, 2, -2, 4, -4, 8, -8, 16, -16, \frac{1}{2}, -\frac{1}{2}.$$

7b.(Source: 3.3.39) Let $q(x) = 2x^4 - 15x^3 + 23x^2 + 24x - 16$. Of the rational numbers found in 7a, -1, $1/2$, and 4 are zeros of $q(x)$. Here's how it looks when we divide by $x+1$ and then by $x - \frac{1}{2}$:

$$\begin{array}{r|rrrr|r}
-1 & 2 & -15 & 23 & 24 & -16 \\
 & & -2 & 17 & -40 & 16 \\
\hline
1/2 & 2 & -17 & 40 & -16 & 0 \\
 & & 1 & -8 & 16 & \\
\hline
 & 2 & -16 & 32 & 0 &
\end{array}$$

Consequently, $q(x) = (x+1)(x-\frac{1}{2})(2x^2-16x+32) = 2(x+1)(x-\frac{1}{2})(x^2-8x+16)$. We can factor the quadratic directly, so the complete factorization is $q(x) = 2(x+1)(x-\frac{1}{2})(x-4)^2$, and the three zeros are -1, $1/2$, and 4 (multiplicity 2).

8.(Source: 3.2.44) $x+2$ is a factor of $p(x)$ if and only if $p(-2) = 0$, so we want $0 = (-2)^3 - k(-2) + 3 = -8 + 2k + 3 = 2k - 5$. Solve to find $k = \frac{5}{2}$. **(done)**

You could also divide $p(x)$ synthetically by $x+2$ to arrive at the same conclusion:

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -k & 3 \\ & & -2 & 4 & 2k-8 \\ \hline & 1 & -2 & 4-k & 2k-5 \end{array}$$

Now solve $2k - 5 = 0$ to find $k = \frac{5}{2}$. **(done)**

9.(Source: 3.3.39) Let $r(x) = 4x^3 + 5x^2 + 2x - 6$. Since the coefficients of r are real, its nonreal zeros come in conjugate pairs, and therefore $x = -1 - i$ is also a zero. The corresponding factors of r are

$$(x+1-i)(x+1+i) = (x+1)^2 - i^2 = x^2 + 2x + 1 + 1 = x^2 + 2x + 2.$$

Find the other factor of r by long division:

$$\begin{array}{r|l} & 4x - 3 \\ \hline x^2+2x+2 & 4x^3 + 5x^2 + 2x - 6 \\ & -(4x^3 + 8x^2 + 8x) \\ \hline & -3x^2 - 6x - 6 \\ & -(-3x^2 - 6x - 6) \\ \hline & 0 \end{array}$$

Therefore $r(x) = (x+1-i)(x+1+i)(4x-3)$, and its zeros are $-1+i$, $-1-i$, and $\frac{3}{4}$.