MATH 111–07 (Kunkle), Exam 4 Name: ____________________

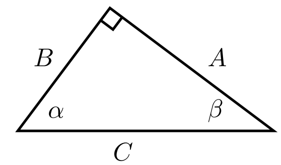
100 pts, 75 minutes Nov 30, 2017

No notes, books, electronic devices, or outside materials of any kind.

Read each problem carefully and simplify your answers.

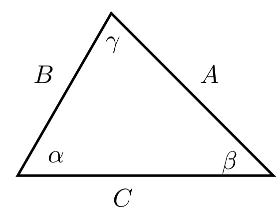
Unless otherwise indicated, supporting work will be required on every problem worth more than 2 points. Unless a problem specifically refers to degrees, use radians in all questions and answers involving angles or trig functions. You are expected to know the values of all trigonometric functions at multiples of $\pi/4$ and of $\pi/6$.

1(7 pts). The parts to this problem refer to the figure at right. The figure is *not* drawn to scale. This means, for instance, that you should not assume that $A > B$ as suggested by the picture.



a. Find C if $\alpha = 80°$ and $B = 11$. b. Find α if $A = 2$ and $B = 3$.

2(15 pts). The parts to this problem refer to the figure at right. The figure is *not* drawn to scale.



a. Find C if $\alpha = 70°$, $\beta = 13°$, and $B = 7$. b. Find γ if $A = 4$, $B = 5$, and $C = 6$.

3(10 pts). Two roads intersect at an acute angle. One car leaves their intersection and travels 3 meters per second along one road. At the same time, another car leaves the intersection and travels 4 meters per second along the other road. If the angle between the paths of the two cars is 80°, how far apart are the cars after 2 seconds?

4a(8 pts). Sketch graph of the function $g(x) = 25 - \left(\frac{1}{5}\right)^x$. Your graph needn't be to scale but should include the coordinates of all intercepts and the equations of any asymptotes.

4b(3 pts). State the domain and range of $g(x)$ from 4a in interval form. Label your answers so I can tell which is which.

5(12 pts). Evaluate the expression.

a. $\log_3 81$ b. $e^{\ln 17}$ c. $e^{-2\ln 3}$ d. $\log_2 10 - \log_2 15 + \log_2 12$

6(4 pts). Find the domain of $h(x) = \ln(4 - x)$. Write your answer in interval form.

7(5 pts). Rewrite the function $\log\left(\dfrac{x^2}{(4-x)^3}\right)$ entirely in terms of logs of linear functions. That is, your answer should not contain the log of a product, quotient, or power.

8(23 pts). Solve for x in the given equation.

a. $\log_5 x = \frac{9}{2} - \log_5 \sqrt{x}$ b. $4^{2-x} = 3^{2x}$ c. $4 \cdot 2^{2x} - 33 \cdot 2^x + 8 = 0$

9. Precalculum-111 is a radioactive substance that loses 10% of its mass every 30 years.

a(8 pts). If I begin with an 80mg sample of Precalculum-111, find an expression for the mass y of my sample after t years.

b(5 pts). Find the half-life of Precalculum-111.

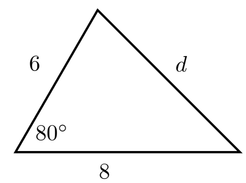
1a.(Source: 4.10.more.1i) $C/11 = \sec 80°$. so $C = 11 \sec 80°$.

1b.(Source: 4.10,more.1o) $\tan\alpha = \frac{2}{3}$, so $\alpha = \tan^{-1}\frac{2}{3}$

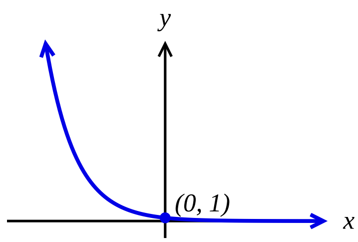
2a.(Source: 4.12.5) (AAS) $\gamma = 180° - \alpha - \beta = 97°$. Now use the Law of Sines: $\dfrac{C}{\sin 97°} = \dfrac{7}{\sin 13°}$, so $C = \dfrac{7\sin 97°}{\sin 13°}$.

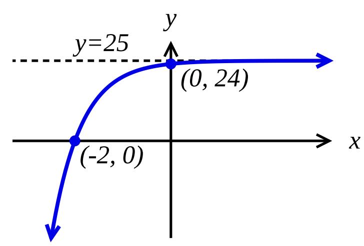
2b.(Source: 4.13.3) (SSS) Use Law of Cosines. $6^2 = 5^2 + 4^2 - 2\cdot 5\cdot 4\cos\gamma$. Solving yields $\gamma = \frac{5}{40} = \frac{1}{8}$, so $\gamma = \cos^{-1}\frac{1}{8}$.

3.(Source: 4.13.more.6) In 2 seconds, the cars have traveled 6 m and 8 m respectively. Let d denote the distance between them. The Law of Cosines tells us that $d^2 = 6^2 + 8^2 - 2\cdot 6\cdot 8\cos 80°$, so $d = \sqrt{100 - 96\cos 80°}$.



4a.(Source: 5.1.9) Start with the (decreasing) graph of $y = (\frac{1}{5})^x$. See graph on the left below. Then reflect across the x-axis and shift up 25 units, as shown in the graph on the right. The new horizontal asymptote is $y = 25$.





To find the y-intercept, set $x = 0$ and calculate $y = 25 - (\frac{1}{5})^0 = 25 - 1 = 24$. To find the x-intercept, set $y = 0$ and solve

$$0 = 25 - \left(\frac{1}{5}\right)^x \quad\Rightarrow\quad \left(\frac{1}{5}\right)^x = 25 \quad\Rightarrow\quad (5^{-1})^x = 5^{-x} = 5^2 \quad\Rightarrow\quad x = -2.$$

4b.(Source: 5.1.20) The domain is the set of x-values we see along the graph, or $(-\infty, \infty)$. (That's because you can evaluate $25 - \left(\frac{1}{5}\right)^x$ at any real number x.) The range is the set of y-values on the graph, or $(-\infty, 25)$.

5a.(Source: 5.2.more.1g) $\log_3 81 = 4$ because $81 = 3^4$.

5b.(Source: 5.2.more.1j) $e^{\ln 17} = 17$, because e^x and $\ln x$ are inverse functions of one another.

5c.(Source: 5.2.21,22) By the properties of logs, $e^{-2\ln 3} = e^{\ln 3^{-2}} = 3^{-2}$, or $\frac{1}{9}$.

5d.(Source: 5.2.more.1h) By properties of logs, $\log_2 10 - \log_2 15 + \log_2 12 = \log_2\left(\frac{10\cdot 12}{15}\right) = \log_2\left(\frac{2\cdot 5\cdot 3\cdot 4}{5\cdot 3}\right) = \log_2 8 = 3$, because $8 = 2^3$.

6.(Source: 5.2.49, 5.2.more.5b) $h(x) = \ln(4-x)$ requires that $4 - x > 0$, or $4 > x$. The domain is $(-\infty, 4)$.

7.(Source: 5.2.more.2g) $\log\left(\dfrac{x^2}{(4-x)^3}\right) = \log x^2 - \log(4-x)^3 = 2\log x - 3\log(4-x)$.

8a.(Source: 5.3.31) $\log_5 x = \frac{9}{2} - \log_5 x^{1/2} \;\Rightarrow\; \log_5 x = \frac{9}{2} - \frac{1}{2}\log_5 x \;\Rightarrow\; \frac{3}{2}\log_5 x = \frac{9}{2} \;\Rightarrow\; \log_5 x = 3 \;\Rightarrow\; x = 5^3 = 125$.

8b.(Source: 5.3.19, 5.3.more.3m) Take ln of both sides and bring down the exponents:

$$\begin{aligned} \ln 4^{2-x} &= \ln 3^{2x} & 2\ln 4 &= 2x\ln 3 + x\ln 4 \\ (2-x)\ln 4 &= 2x\ln 3 & 2\ln 4 &= x(2\ln 3 + \ln 4) \\ 2\ln 4 - x\ln 4 &= 2x\ln 3 & x &= \frac{2\ln 4}{2\ln 3 + \ln 4} \end{aligned}$$

You could also write this as $\frac{\ln 16}{\ln 36}$. It is also correct to replace the ln throughout with a log of a base other than e. If you used base 4 or base 3, you get x equal $\frac{2}{1+2\log_4 3}$ or $\frac{2\log_3 4}{2+\log_3 4}$.

8c.(Source: 5.3.more.2h) Factor $4\cdot 2^{2x} - 33\cdot 2^x + 8 = (4\cdot 2^x - 1)(2^x - 8) = 0$, which implies 2^x equals either $\frac{1}{4}$ or 8, which means $x = -2$ or $x = 3$.

9a.(Source: 5.4.more.5) The mass y of the sample decays exponentially, so $y = y_0 e^{kt}$, where y_0 equals the value of y at $t = 0$, or 80. To find k, set $t = 30$ and $y = 72$ (that is, 90% of 80) and solve:

$$80e^{30k} = 72 \;\Rightarrow\; e^{30k} = \frac{72}{80} = 0.9 \;\Rightarrow\; 30k = \ln(0.9) \;\Rightarrow\; k = \frac{1}{30}\ln(0.9)$$

and so, $y = 80e^{t\frac{1}{30}\ln(0.9)}$.

9b. To find the half-life of Precalculum-111, you can set $y = 40$ and solve for t:

$$80e^{t\frac{1}{30}\ln(0.9)} = 40$$

or, equivalently,

$$e^{t\frac{1}{30}\ln(0.9)} = 0.5 \;\Rightarrow\; \frac{t}{30}\ln(0.9) = \ln(0.5) \;\Rightarrow\; t = \frac{30\ln(0.5)}{\ln(0.9)}$$