No notes, books, electronic devices, or outside materials of any kind.

Read each problem carefully and simplify your answers.

Unless otherwise indicated, supporting work will be required on every problem worth more than 2 points. Unless a problem specifically refers to degrees, use radians in all questions and answers involving angles or trig functions. You are expected to know the values of all trigonometric functions at multiples of $\pi/4$ and of $\pi/6$.

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1(12 pts). Find the values of the given trig functions at the given values of $t$. Write "DNE" when appropriate. Supporting work not required on this problem.

| $t$ | $\frac{\pi}{6}$ | $\frac{23\pi}{4}$ | $-\frac{4\pi}{3}$ | $-\frac{7\pi}{2}$ | $-\frac{3\pi}{4}$ | $3\pi$ |
|---|---|---|---|---|---|---|
| $\sin t$ | | | | | | |
| $\cos t$ | | | | | | |
| $\tan t$ | | | | | | |
| $\csc t$ | | | | | | |

2(14 pts). Suppose that $\sin x = \frac{2}{3}$ and that $\frac{\pi}{2} < x < \frac{3\pi}{2}$. Find the following.

a. $\cos x$ b. $\cot x$ c. $\sin(2x)$ d. $\cos(x + \frac{\pi}{6})$

3(3 pts). Convert $40°$ to radians.

4(3 pts). Convert $\frac{2\pi}{5}$ radians to degrees.

5(15 pts). Sketch the graph of $r(x) = \frac{(x^2+4)(x+1)}{(x+1)(x-2)}$.

Find and report all intercepts, asymptotes, and holes, or state that they do not exist. Give asymptotes by their equations; give holes and intercepts by their coordinates.

6(13 pts). Sketch one cycle of the graph of $y = -1 + 3\sin\left(2x - \frac{\pi}{4}\right)$. Draw the axes where you wish and label hashmarks so as to clearly indicate every point in your cycle where the sine equals 0, 1, or $-1$. You are not required to find the intercepts of this graph.

What is the period of this function?

7(8 pts). Sketch one cycle of the graph of $y = -\tan(\frac{1}{2}x)$. Draw the axes where you wish. Use and label the hashmarks so that your graph clearly shows the locations of all asymptotes and intercepts that occur on your cycle.

What is the period of this function?

8(18 pts). Find the following.

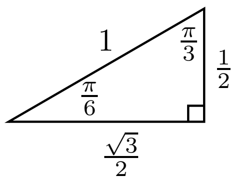
a. $\cos\left(\frac{3\pi}{8}\right)$ b. $\sin^{-1}\left(-\frac{1}{2}\right)$ c. $\tan^{-1} 1$

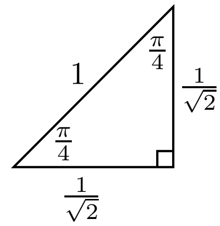
d. $\tan^{-1}\left(\tan\left(\frac{4\pi}{5}\right)\right)$ e. $\sin\left(\cos^{-1}\left(-\frac{4}{5}\right)\right)$ f. $\cos\left(\cos^{-1}\left(-\frac{4}{5}\right)\right)$

9(14 pts). Find all solutions $x$ to the given equation.

a. $\sec(2x) = \sqrt{2}$ b. $\sin^2 x + 3\sin x + 2 - \cos^2 x = 0$

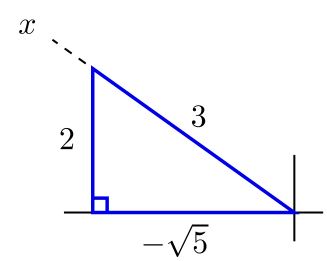
1.(Source: 4.2.more.1, 4.4.1,2) Determine the reference number of each angle and the quadrant containing its terminal point. Draw a vertical line from the terminal point to the $x$-axis and a line from the terminal point to the origin. Look for these two triangles. You were not required to rationalize denominators.




| $t$ | $\frac{\pi}{6}$ | $\frac{23\pi}{4}$ | $-\frac{4\pi}{3}$ | $-\frac{7\pi}{2}$ | $-\frac{3\pi}{4}$ | $3\pi$ |
|---|---|---|---|---|---|---|
| quadrant | $I$ | $IV$ | $II$ | North Pole | $III$ | West Pole |
| ref. number | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\frac{\pi}{4}$ | 0 |
| $\sin t$ | $1/2$ | $-1/\sqrt{2}$ | $\sqrt{3}/2$ | 1 | $-1/\sqrt{2}$ | 0 |
| $\cos t$ | $\sqrt{3}/2$ | $1/\sqrt{2}$ | $-1/2$ | 0 | $-1/\sqrt{2}$ | $-1$ |
| $\tan t = \frac{\sin t}{\cos t}$ | $1/\sqrt{3}$ | $-1$ | $-\sqrt{3}$ | DNE | 1 | 0 |
| $\csc t = \frac{1}{\sin t}$ | 2 | $-\sqrt{2}$ | $2/\sqrt{3}$ | 1 | $-\sqrt{2}$ | DNE |

2.(Source: 4.2.3, 4.4.23, 4.5.29, 4.5.more.1a) Since $\frac{\pi}{2} < x < \frac{3\pi}{2}$ and $\sin x > 0$, we know that $x$ is in Quadrant III. Use $\sin x = 2/3$ to label the vertical and hypotenuse of the triangle in Quadrant III. Find the missing side by the Pythagorean theorem:




a. $\cos x = -\sqrt{5}/3$ b. $\cot x = \dfrac{\cos x}{\sin x} = \dfrac{-\sqrt{5}/3}{2/3} = -\sqrt{5}/2$

c. $\sin(2x) = 2\sin x \cos x = 2\left(\frac{2}{3}\right)\left(\frac{-\sqrt{5}}{3}\right) = \frac{-4\sqrt{5}}{9}$

d. $\cos(x + \frac{\pi}{6}) = \cos x \cos(\frac{\pi}{6}) - \sin x \sin(\frac{\pi}{6}) = -\frac{\sqrt{5}}{3} \cdot \frac{\sqrt{3}}{2} - \frac{2}{3} \cdot \frac{1}{2} = \frac{-\sqrt{15}-2}{6}$

3.(Source: 4.1.25) $40° = 40° \left(\frac{\pi}{180°}\right) = \frac{2\pi}{9}$ radians.

4.(Source: 4.1.33) $\frac{2\pi}{5} \cdot \frac{180°}{\pi} = 72°$.

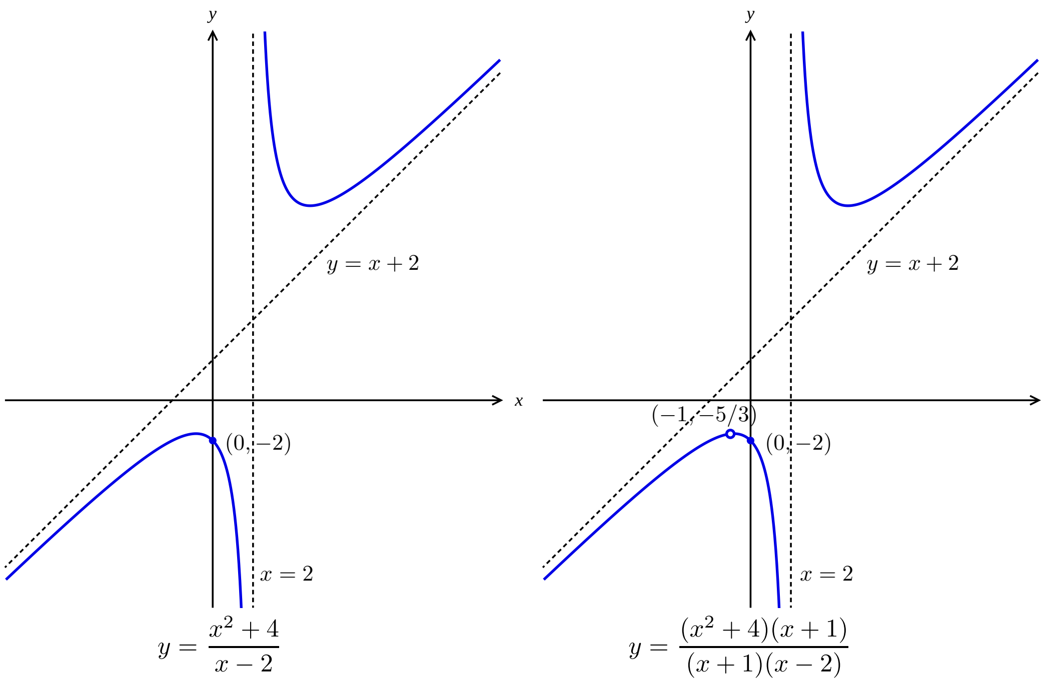
5.(Source: 3.5.more.1v) The common factor $x + 1$ means the graph of $r(x)$ is the same as that of $\frac{x^2+4}{x-2}$, except that $y = r(x)$ is undefined at $x = -1$. Now graph $y = \frac{x^2+4}{x-2}$ There's a vertical asymptote at $x = 2$, where the denominator is zero. Since the degree of the numerator is one more than that of the denominator, there is a slant asymptote, which we can find by long division, or, more simply, by synthetic division:

$$\begin{array}{r|rrr} 2 & 1 & 0 & 4 \\ & & 2 & 4 \\ \hline & 1 & 2 & 8 \end{array}$$

The slant asymptote is the graph of the quotient, or $y = x + 2$.

At $x = 0$ we calculate $y = 4/(-2) = -2$, so the $y$-intercept is $(0, -2)$. Setting $y = 0$ leads to $x^2 + 4 = 0$, which has no real solutions, so there is no $x$-intercept.
To graph, it helps to make a sign chart:

$$
\begin{array}{r c}
x^2 + 4 : & + + + + + + + + + + + \\
x - 2 : & - - - - - - 0 + + + + + + \\
\dfrac{x^2+4}{x-2} : & - - - - - \mathrm{VA} + + + + + + \\
\hline
x : & 2
\end{array}
$$



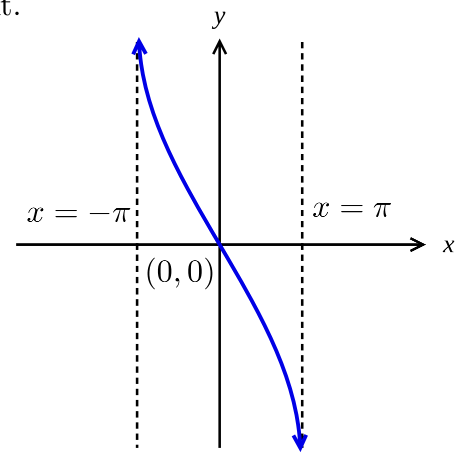

Combining all the information we have generates a graph for $y = \frac{x^2+4}{x-2}$. Punch out a hole from this graph at $x = -1$ to obtain the graph of $y = \frac{(x^2+4)(x+1)}{(x+1)(x-2)}$. The $y$-coordinate of the hole is the altitude of $y = \frac{x^2+4}{x-2}$ at $x = -1$, or $(1 + 4)/(-1 - 2) = -5/3$.

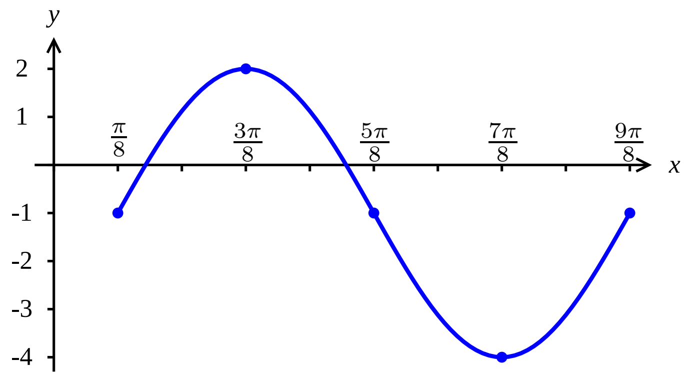
6.(Source: 4.3.31-36,43) The function will go through one cycle when the angle inside the sine goes from 0 to $2\pi$.

$$0 \le 2x - \frac{\pi}{4} \le 2\pi$$
$$\frac{\pi}{4} \le 2x \le 2\pi + \frac{\pi}{4} = \frac{9\pi}{4}$$
$$= \frac{\pi}{8} = \frac{1}{2} \cdot \frac{\pi}{4} \le x \le \frac{1}{2} \cdot \frac{9\pi}{4} = \frac{9\pi}{8}$$

So, we'll draw a cycle of the sine starting at $x = \frac{\pi}{8}$ and ending at $x = \frac{9\pi}{8}$. The amplitude is 3, and the curve is shifted down 1 unit, so that its maximum and minimum altitudes are 2 and -4. The five basic points to the graph: $(\frac{\pi}{8}, -1)$, $(\frac{3\pi}{8}, 2)$, $(\frac{5\pi}{8}, -1)$, $(\frac{7\pi}{8}, -4)$, and

$(\frac{9\pi}{8}, -1)$. The period is the horizontal length of the cycle, or $\frac{9\pi}{8} - \frac{\pi}{8} = \pi$. See graph below right.




7.(Source: 4.4.30,32) The function will go through one cycle when the angle inside the tangent goes from $-\pi/2$ to $\pi/2$:

$$-\frac{\pi}{2} \leq \frac{1}{2}x \leq \frac{\pi}{2} \quad \Longrightarrow \quad -\pi \leq x \leq \pi$$

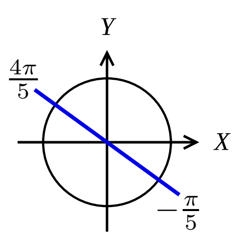
The period is the length of this interval, or $2\pi$. The coefficient $-1$ causes graph of the tangent to be reflected across the $x$-axis. (Because $\tan x$ is an odd function, this happens to be the same as reflecting its graph across the $y$-axis.) See graph above left.

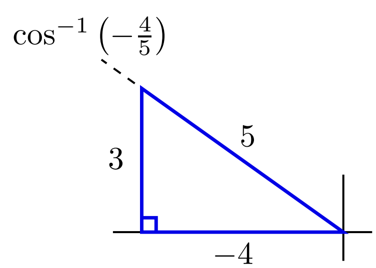
8a.(Source: 4.5.37) By the half-angle indentity, $\cos^2\left(\frac{3\pi}{8}\right) = \frac{1}{2}\left(1 + \cos\left(2 \cdot \frac{3\pi}{8}\right)\right) = \frac{1}{2}\left(1 + \cos\left(\frac{3\pi}{4}\right)\right) = \frac{1}{2}(1 - \frac{1}{\sqrt{2}})$. Although $\frac{3\pi}{4}$ is in Quadrant II, $\frac{3\pi}{8}$ is in Quadrant I. Its cosine is therefore positive and we take the positive square root: $\cos\left(\frac{3\pi}{8}\right) = \sqrt{\frac{1}{2}(1 - \frac{1}{\sqrt{2}})}$

8b.(Source: 4.7.more.2g) $\sin^{-1}\left(-\frac{1}{2}\right)$ is the angle in $[-\frac{\pi}{2}, \frac{\pi}{2}]$ whose sine is $-\frac{1}{2}$. That's $-\frac{\pi}{6}$.

8c.(Source: 4.7.more.3c) $\tan^{-1} 1$ is the angle in $(-\frac{\pi}{2}, \frac{\pi}{2})$ whose tangent is 1. That's $\frac{\pi}{4}$.

8d.(Source: 4.7.more.6b) $\tan(\frac{4\pi}{5})$ is the slope the terminal side of $\frac{4\pi}{5}$, and $\tan^{-1}(\tan(\frac{4\pi}{5}))$ is the angle $\theta$ in $(-\frac{\pi}{2}, \frac{\pi}{2})$ for which $\tan\theta = \tan(\frac{4\pi}{5})$. To have the same slope, the terminal sides of $\theta$ and of $\frac{4\pi}{5}$ must form a line, so $\theta$ must be $\frac{-\pi}{5}$. (See the figure on the left below.)

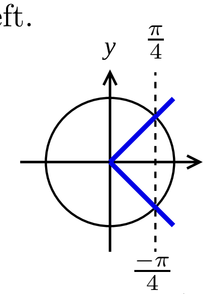


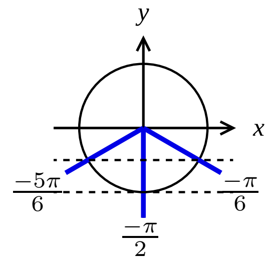

8e.(Source: 4.7.more.4f) $\cos^{-1}\left(-\frac{4}{5}\right)$ is the angle in $[0, \pi]$ whose cosine is $-\frac{4}{5}$. Since its cosine is negative, we know that $\cos^{-1}\left(-\frac{4}{5}\right)$ is in Quadrant III. Use $\cos x = -4/5$ to label the horizontal and hypotenuse of the triangle in Quadrant III. (See the figure on the right above.) Find the missing side ($= 3$) by the Pythagorean theorem. Then sine of this angle is $y/r = 3/5$.

8f.(Source: 4.7.more.7g) $\cos\left(\cos^{-1}\left(-\frac{4}{5}\right)\right) = -\frac{4}{5}$ by the definition of the $\cos^{-1}$.

9a.(Source: 4.8.17) Secant is the reciprocal of cosine, so $\sec(2x) = \sqrt{2}$ means $\cos(2x) = 1/\sqrt{2}$. The angle $2x$ must equal $\pm\frac{\pi}{4} + 2\pi n$, so $x = \pm\frac{\pi}{8} + \pi n$, for any integer $n$. See figure below on the left.



9a. $\cos 2x = 1/\sqrt{2}$



9b. $\sin x = -1$ or $-1/2$.

9b.(Source: 4.8.27) Replace $\cos^2 x$ with $1 - \sin^2 x$ to obtain

$$0 = \sin^2 x + 3\sin x + 2 - (1 - \sin^2 x) = 2\sin^2 x + 3\sin x + 1 = (2\sin x + 1)(\sin x + 1).$$

So either $\sin x = -1/2$ or $\sin x = -1$. That makes $x$ an angle that intersects the unit circle at altitude $-1$ or $-1/2$. See figure above on the right. Solutions are

$$x = -\pi/2 + 2\pi n \qquad x = -\pi/6 + 2\pi n \qquad x = -5\pi/6 + 2\pi n.$$