No notes, books, electronic devices, or outside materials of any kind.

Read each problem carefully and simplify your answers.

Unless otherwise indicated, supporting work will be required on every problem worth more than 2 points.

1(4 pts). Identify the polynomial function as even, odd, or neither of these. No supporting work is required on this problem.

a. $x^4 + x - 1$ b. $-x^5 + 2x^3 - x$ c. $-3x^4 - 4x^2 + 2$ d. $4x^5(x^2 - 1)$

2(12 pts). Compute the difference quotient $\frac{f(x+h)-f(x)}{h}$ if $f(x) = \frac{3}{3-2x}$. Find and cancel the common factor h from top and bottom.

3(9 pts). Find the quotient and remainder when $2x^3 - x^2 + 2x - 5$ is divided by $(x-1)^2$. Label your answers so I can tell which is which.

4a(8 pts). Find the two functions defined implicitly by the equation $x^2 - xy + y^2 = 12$. Write your answer in the form "$f(x) = \cdots$ and $g(x) = \cdots$."

4b(4 pts). The two functions in 4a have the same domain. State it in interval form.

5(7 pts). Write the complex number in the form $a + bi$:

a. $(7 + 3i) - (2 - 5i)$ b. $(7 + 3i)(2 - 5i)$

6(11 pts). Let $f(x) = (x-2)^{-5/3} - 3$.

a. Find $f^{-1}(x)$.

b. State the domain and range of f^{-1} in interval form. Label them so I can tell which is which.

7(10 pts). Sketch the graph of $-3(x-1)(x+1)^3(x-3)^2$. Your graph needn't be to accurate scale but should show all intercepts, the graph's behavior at each, and the correct end behavior of the function.

8(12 pts). A rectangular plot of land is surrounded by fence and divided into two pens by an additional length of fence parallel to one side of the rectangle. If the total length of fence is 4000m, express the total area of the plot as a function of the length of one side of the rectangle.

9a(6 pts). List all the possible rational zeros of $p(x) = 2x^4 + 5x^3 + ax^2 + bx + 4$ if a and b are integers.

9b(17 pts). Factor completely and find all zeros of $q(x) = 2x^4 + 5x^3 - 7x^2 - 6x + 4$.

1.(Source: 3.1.9-12) A polynomial is even (odd) iff its expanded form $ax^n+bx^{n-1}+\cdots+cx+d$ consists entirely of even (odd) monomials.
a. x^4+x-1 is neither even nor odd. b. $-x^5+2x^3-x$ is odd. c. $-3x^4-4x^2+2$ is even.
d. $4x^5(x^2-1)=4x^7-4x^5$ is odd.

2.(Source: 2.10.20) $\dfrac{f(x+h)-f(x)}{h} = \dfrac{\frac{3}{3-2(x+h)}-\frac{3}{3-2x}}{h} =$

$$= \frac{1}{h}\left[\left(\frac{3-2x}{3-2x}\right)\left(\frac{3}{3-2x-2h}\right)-\left(\frac{3}{3-2x}\right)\left(\frac{3-2x-2h}{3-2x-2h}\right)\right]$$
$$= \frac{1}{h}\left[\frac{3(3-2x)-3(3-2x-2h)}{(3-2x)(3-2x-2h)}\right] = \frac{1}{h}\left[\frac{9-6x-9+6x+6h}{(3-2x)(3-2x-2h)}\right]$$
$$= \frac{1}{h}\left[\frac{6h}{(3-2x)(3-2x-2h)}\right] = \frac{6}{(3-2x)(3-2x-2h)}$$

3.(Source: 3.2.5) Expand the divisor: $(x-1)^2 = x^2-2x+1$. Then do long division.

$$\begin{array}{r|l}
 & \quad 2x+3 \\ \hline
x^2-2x+1 & 2x^3 - x^2 + 2x - 5 \\
 & -(2x^3-4x^2+2x) \\ \hline
 & \qquad 3x^2 \qquad - 5 \\
 & \quad -(3x^2-6x+3) \\ \hline
 & \qquad\qquad 6x-8
\end{array}$$

The quotient is $2x+3$, and the remainder is $6x-8$.

4.(Source: 2.7.23) To solve for y, get zero on one side and use the quadratic formula. Treating y as the unknown, the y^2-coefficient, the y-coefficient, and the constant term are, respectively, $a=1$, $b=-x$, and $c=x^2-12$:

$$y^2-xy+x^2-12=0 \Longrightarrow y = \frac{x\pm\sqrt{(-x)^2-4(x^2-12)}}{2} = \frac{x\pm\sqrt{48-3x^2}}{2}.$$

The two functions are $f(x)=\frac{1}{2}(x+\sqrt{48-3x^2})$ and $g(x)=\frac{1}{2}(x-\sqrt{48-3x^2})$.
Their domain is the solution set to $0\le 48-3x^2$. Dividing both sides by 3 doesn't change the direction of the inequality, so $0\le 16-x^2=(4-x)(4+x)$. Here's a sign chart:

$$\begin{array}{r|l}
4+x: & ----0+++++++++++ \\
4-x: & ++++++++++0----- \\
(4+x)(4-x): & ----0+++++0----- \\ \hline
x: & \qquad -4 \qquad\qquad 4
\end{array}$$

The domain is the solution set, $[-4,4]$.

5a.(Source: 3.3.3) $(7+3i)-(2-5i)=7+3i-2+5i=5+8i$.

5b. Use FOIL. $(7+3i)(2-5i)=14-35i+6i-15i^2=14-29i-15(-1)=29-29i$.

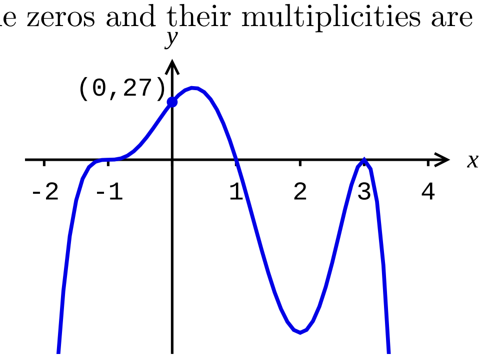
6.(Source: 2.8.more.1tw) Set $y=(x-2)^{-5/3}-3$ and solve for x:

$$y+3=(x-2)^{-5/3} \quad\Longrightarrow\quad (y+3)^{-3/5}=x-2 \quad\Longrightarrow\quad x=2+(y+3)^{-3/5}=f^{-1}(y),$$

so $f^{-1}(x) = 2 + (x+3)^{-3/5}$.
$(x+3)^{-3/5} = \frac{1}{(x+3)^{3/5}}$, so the only condition on x for $f^{-1}(x)$ to be defined is that $x \neq -3$. That is, the domain of f^{-1} is $(-\infty, -3) \cup (-3, \infty)$.
The range of f^{-1} is the domain of $f(x) = (x-2)^{-5/3} - 3$, and, because of the negative exponent, this is all numbers except 2: $(\infty, 2) \cup (2, \infty)$.

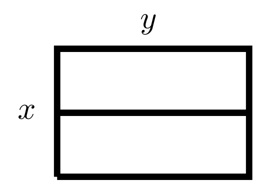
7.(Source: 3.1.more.1b) $-3(x-1)(x+1)^3(x-3)^2 = -3x^6+$ lower order terms, so the end behavior of this polynomial is $y \to -\infty$ as $x \to \pm\infty$. The zeros and their multiplicities are

zero	-1	1	3
multiplicity	3	1	2



y changes signs at its zeros of odd multiplicity, -1 and 1. The graph flattens out (is tangent to the x-axis) at the zeros of multiplicity greater than 1, that is, at $x = -1$ and 3. Plug in $x = 0$ to calculate the y-intercept: $-3(0-1)(0+1)^3(0-3)^2 = 27$. Here's a nice graph:

8.(Source: 2.9.32) Call the lengths of the two sides x and y, say. Then the area of the rectangle is $A = xy$. The total length of fence is $2x + 3y = 4000$. Solving, $y = (4000 - 2x)/3$. Plug this in for y and get $A = x(4000 - 2x)/3$.
(If you solve for x instead of y and wrote A in terms of y, then you'll have $A = y(4000 - 3y)/2$. Either answer is correct.)



9a.(Source: 3.4.more.1w) The numerator of a rational zero must divide the constant term, 4, and the denominator must divide the lead coefficient 2. Since $\frac{2}{2} = 1$ and $\frac{4}{2} = 2$, the possible rational zeros are

$$1, -1, 2, -2, 4, -4, \frac{1}{2}, -\frac{1}{2}.$$

9b. Of the rational numbers found in 9a, 1/2 and -1 are zeros. Here's how it looks when we divide by $x + 1$ and then by $x - \frac{1}{2}$:

$$\begin{array}{r|rrrr|r}
-1 & 2 & 5 & -7 & -6 & 4 \\
 & & -2 & -3 & 10 & -4 \\
\hline
1/2 & 2 & 3 & -10 & 4 & 0 \\
 & & 1 & 2 & -4 & \\
\hline
 & 2 & 4 & -8 & 0 &
\end{array}$$

Consequently, $q(x) = (x+1)(x-\frac{1}{2})(2x^2+4x-8) = 2(x+1)(x-\frac{1}{2})(x^2+2x-4)$. The remaining quadratic doesn't factor over the integers, but we can find the zeros by other means—the quadratic formula, say—and then figure out the factors. We can also put the quadratic in standard form and then factor using the square minus a square formula. To illustrate, $x^2 + 2x - 4 = x^2 + 2x + 1 - 4 - 1 = (x+1)^2 - 5 = (x+1-\sqrt{5})(x+1+\sqrt{5})$. Setting this equal zero gives the last two zeros, and so the factorization is

$$q(x) = 2(x+1)(x-\tfrac{1}{2})(x+1-\sqrt{5})(x+1+\sqrt{5}),$$

and the four zeros are -1, $1/2$, and $-1 \pm \sqrt{5}$.