No notes, books, electronic devices, or outside materials of any kind.

Read each problem carefully and simplify your answers.

Unless otherwise indicated, supporting work will be required on every problem worth more than 2 points.

1(25 pts). Solve the inequalities. Write the solution set in interval notation.

a. $|4x - 7| \geq 17$ b. $x^3 - 16x \geq 0$ c. $\frac{x+2}{x+3} \leq 2$

2(7 pts). Find an equation of the circle that is centered at $(3, -1)$ and passing through the point $(-2, -4)$.

3(5 pts). Find an equation of the line passing through the points $(-3, 7)$ and $(1, 2)$.

4(7 pts). Find an equation of the line passing through the point $(-3, 7)$ and perpendicular to $3x + 2y = 5$.

5(10 pts). Use factorization, binomial expansion, or rationalization to find and cancel the common factor in each expression.

a. $\dfrac{x-3}{\sqrt{x+1}-2}$ b. $\dfrac{12 - 3(h+2)^2}{h}$

6(16 pts). Graph the function. Find all intercepts and asymptotes, if any, and include them in your graph. State the domain and range of the functions **in parts b and d only**.

a. $f(x) = \sqrt{x}$ b. $h(x) = \sqrt{3-x}$ c. $g(x) = x^{4/3}$ d. $k(x) = x^{4/3} - 16$

7(6 pts). Rewrite the expression $|x+1| - |x-2|$ without absolute value symbols if $-1 \leq x < 2$.

8(14 pts). Let $f(x) = \sqrt{1+x}$ and $g(x) = x^2 - 10$. Find each of the following functions and their domain. Simplify your answers and express the domains in interval form.

a. $\big(f \circ g\big)(x)$ b. $\big(g \circ f\big)(x)$

9(6 pts). Find all x- and y-intercepts of the graph of $p(x) = \begin{cases} -2x - 1 & \text{if } x \leq 0 \\ x + 5 & \text{if } x > 0 \end{cases}$.
Label your answers so I can tell which is which.

10. Let $q(x) = -2x^2 + 20x - 32$. Work in the space below and label your answers to the following questions.

a(5 pts). Rewrite $q(x)$ in standard form.
b(6 pts). Find the coordinates of the vertex and all intercepts.
c(2 pts). Give the range of $q(x)$ in interval notation.

1a.(Source: 1.2.42) First solve the related equation: $|4x-7|=17$ means

$$\begin{aligned} 4x-7&=17 & 4x-7&=-17\\ 4x&=24 & 4x&=-10\\ x&=6 & x&=-5/2 \end{aligned}$$

These solutions divide the x-axis into three intervals. Test the inequality on each. Here's what I found when I did this:

$$\begin{array}{c|ccccc} |4x-7|\geq 17: & \text{T} & & \text{F} & & \text{T}\\ \hline x: & & -\frac{5}{2} & & 6 & \end{array}$$

Including the endpoints (since the inequality is true at these), the solution set is $(\infty,-5/2]\cup[6,\infty)$.

1b.(Source: 1.1.35,1.1.43) Factor and make a sign chart:

$$x^3-16x=x(x^2-16)=x(x-4)(x+4)\geq 0.$$

$$\begin{array}{r|l} x-4: & ------------------0+++++++\\ x: & ----------0+++++++++++++\\ x+4: & ------0+++++++++++++++++\\ x(x-4)(x+4): & -----0++++0-----0++++++\\ \hline x: & \qquad\qquad -4 \qquad\qquad 0 \qquad\qquad 4 \end{array}$$

Solution set is $[-4,0]\cup[4,\infty)$.

1c.(Source: 1.1.52,more.1i) Get a zero on one side, factor the other, and then make a sign chart:

$$\frac{x+2}{x+3}-2\leq 0 \quad\Longrightarrow\quad \frac{x+2-2(x+3)}{x+3}\leq 0 \quad\Longrightarrow\quad \frac{-x-4}{x+3}\leq 0$$

$$\begin{array}{r|l} -x-4: & ++++++++0--------------------\\ x+3: & ----------------------0+++++++++\\ \dfrac{-x-4}{x+3}: & -----------0+++++++++\text{DNE}--------\\ \hline x: & \qquad\qquad\qquad -4 \qquad\qquad\qquad -3 \end{array}$$

Solution set is $(-\infty,-4]\cup(-3,\infty)$

2.(Source: 1.4.21) The radius of the circle is the distance from $(3,-1)$ to $(-2,-4)$, or $\sqrt{5^2+3^2}=\sqrt{34}$. The equation of the circle is $(x-3)^2+(y+1)^2=34$.

3.(Source: 2.3.23) Slope $=\frac{\Delta \mathsf{x}}{\Delta \mathsf{y}}=\frac{7-2}{-3-1}=-\frac{5}{4}$. In point-slope form, the line can be written $y-7=\frac{5}{4}(x+3)$, or $y-2=-\frac{5}{4}(x-1)$.

4.(Source: 2.3.33) First find the slope of $3x+2y=5$ by converting to slope-intercept form: $2y=-3x+5 \quad\Longrightarrow\quad y=-\frac{3}{2}x+\frac{5}{2}$. This line has slope $-\frac{3}{2}$, so the desired line must have slope $\frac{2}{3}$. Its equation is $y-7=\frac{2}{3}(x+3)$.

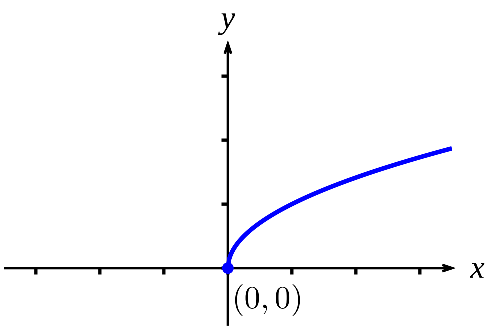
5a.(Source: 1.5.30) Rationalize the denominator:

$$\frac{(x-3)}{(\sqrt{x+1}-2)}\cdot\frac{(\sqrt{x+1}+2)}{(\sqrt{x+1}+2)}=\frac{(x-3)(\sqrt{x+1}+2)}{\sqrt{x+1}^2-2^4}=\frac{(x-3)(\sqrt{x+1}+2)}{x+1-4}$$
$$=\frac{(x-3)(\sqrt{x+1}+2)}{x-3}=\sqrt{x+1}+2$$

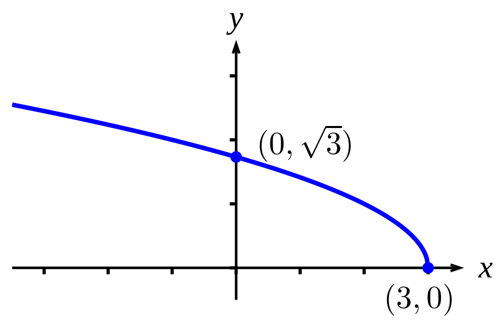
5b.(Source: 1.5.14) Expand the binomial:

$$\frac{12-3(h+2)^2}{h}=\frac{12-3(h^2+4h+4)}{h}=\frac{12-3h^2-12h-12)}{h}$$
$$=\frac{-3h^2-12h}{h}=\frac{h(-3h-12)}{h}=-3h-12$$

6.(Source: 2.2.more.1o) To obtain the graph of $y=\sqrt{3-x}$, reflect $y=\sqrt{x}$ across the y-axis, and shift the result right 3 units. To see this, it helps to find the domain and intercepts of $\sqrt{3-x}$ algebraically. This function requires $0\le 3-x$, or $x\le 3$, so its domain is $(-\infty,3]$. To find the y-intercept, set $x=0$ and calculate $y=\sqrt{3-0}=\sqrt{3}$. To find the x-intercept, set $y=0$ and solve for x: $\sqrt{3-x}=0\Rightarrow 3-x=0^2\Rightarrow 3=x$. Shifting right didn't change the range of the square root: $[0,\infty)$.

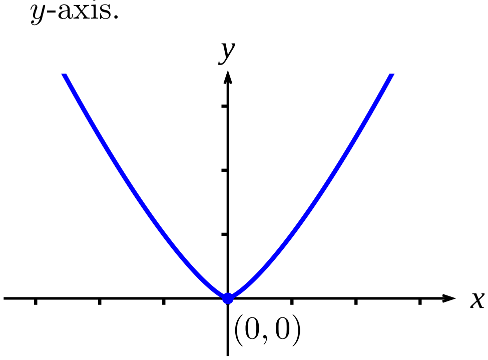


a. $y=\sqrt{x}$

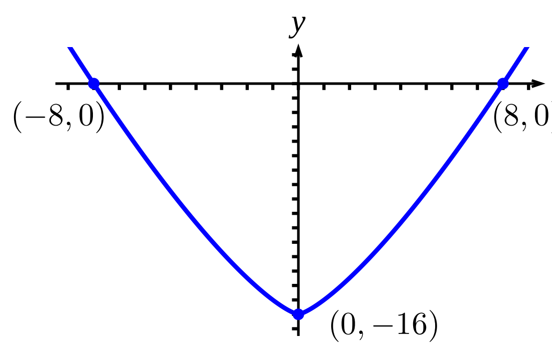


b. $y=\sqrt{3-x}$

In quadrant 1, the graph of $x^{4/3}$ looks like that of x^2, since $4/3>1$. This function equals $(\sqrt[3]{x})^4$, which is defined for $x<0$ and is even. Therefore its graph is symmetric across the y-axis.



c. $y=x^{4/3}$



d. $y=x^{4/3}-16$

The graph of $x^{4/3} - 16$ is obtained by shifting that of $x^{4/3}$ downward 16 units, making the new y-intercept $(0, -16)$. To find the x-intercept, set $y = 0$ and solve $0 = x^{4/3} - 16$:

$$x^{4/3} = 16 \qquad x = \pm 16^{3/4} =^{*} \pm(\sqrt[4]{16})^3 = \pm 2^3 = \pm 8.$$

The x-intercepts are $(8, 0)$ and $(-8, 0)$. Avoid working with large numbers. After the step marked *, I took the cube root and then raised the result to the power 4. If you took the 4th power before taking the cube root, you were left having to calculate $\sqrt[4]{4096}$.
The domain and range of $x^{4/3}$ are $(-\infty, \infty)$ and $[0, \infty)$, respectively. Because of the vertical shift, the domain and range of $x^{4/3} - 16$ are $(-\infty, \infty)$ and $[-16, \infty)$, respectively.

7.(Source: 1.2.17)

$$|x+1| = \begin{cases} x+1 & \text{if } x+1 \geq 0 \\ -(x+1) & \text{if } x+1 \leq 0 \end{cases} \qquad |x-2| = \begin{cases} x-2 & \text{if } x-2 \geq 0 \\ -(x-2) & \text{if } x-2 \leq 0 \end{cases}$$

If $-1 \leq x \leq 2$, then $0 \leq x+1$ and $x-2 \leq 0$, and so

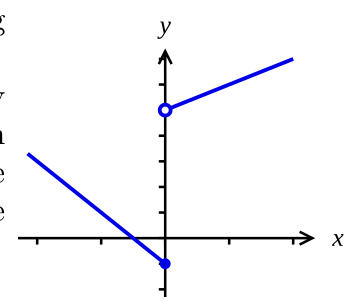
$$|x+1| - |x-2| = x + 1 - (-(x-2)) = x + 1 + x - 2 = 2x - 1.$$

8a. (Source: 2.6.more.1d,2d) $\big(f \circ g\big)(x) = \sqrt{1 + (x^2 - 10)} = \sqrt{x^2 - 9}$. The domain is the solution set to $0 \leq x^2 - 9 = (x-3)(x+3)$, which we find with a sign chart.

$x+3:$	$- - - - - - - - -$ 0	$+ + + + + + + + + +$	$+ + + + + + + +$
$x-3:$	$- - - - - - - - -$	$- - - - - - - - - -$ 0	$+ + + + + + + +$
$x^2-9:$	$+ + + + + + + + +$ 0	$- - - - - - - - - -$ 0	$+ + + + + + + +$
$x:$	-3	3	

Hence the domain of $\big(f \circ g\big)(x)$ is $(-\infty, -3] \cup [3, \infty)$.
8b. $\big(g \circ f\big)(x) = (\sqrt{1+x})^2 - 10$, which, when defined, simplifies to $x - 9$. For this function to be defined, the radical requires that $1 + x \geq 0$, or $x \geq -1$. Hence the domain is $[-1, \infty)$.

9. (Source: 2.5.1-4,7-8) Find the y-intercept by evaluating $p(x)$ at $x = 0$. Since the first case applies, $p(0) = -2 \cdot 0 - 1 = -1$, and therefore the y-intercept is $(0, -1)$. Some students said that the graph had two y-intercepts, an impossibility along the graph of a function.
When we set $-2x - 1 = 0$ and $x + 5 = 0$ we find that the only possible x-intercepts are $x = -1/2$ and $x = -5$. The first is an intercept, since $x = -1/2$ is on that part of the domain where $p(x)$ is given by $-2x - 1$. But since $x = -5 \leq 0$, we do not use $x + 5$ to calculate $p(-5)$. Only $-1/2$ is an x-intercept.



10a. (Source: 2.4.7-20,more.1j) $q(x) = -2(x^2 - 10x) - 32 = -2(x^2 - 10x + 25) - 32 + 50 = -2(x-5)^2 + 18$.

10b. Vertex is $(5, 18)$. When $x = 0$, calculate $y = -32$, so the y-intercept is $(0, -32)$.
Set $y = 0$ and solve for x, using the standard form.

$$\begin{aligned} -2(x-5)^2 + 18 &= 0 \\ 2(x-5)^2 &= 18 \\ (x-5)^2 &= 9 \end{aligned} \qquad \begin{aligned} x - 5 &= \pm 3 \\ x &= 5 \pm 3 = 8 \text{ or } 2 \end{aligned}$$

so the x-intercepts are $(8, 0)$ and $(2, 0)$. We could also have solved $q(x) = 0$ by factoring: $-2x^2 + 20x - 32 = -2(x-8)(x-2)$.

10c. The graph of q opens downward, so 18 is its maximum value. The range is $(-\infty, 18]$.
You were not required to sketch the graph of $q(x)$, but here's what it looks like: