

No notes, books, electronic devices, or outside materials of any kind.

Read each problem carefully and simplify your answers.

Unless otherwise indicated, supporting work will be required on every problem worth more than 2 points.

1(4 pts). Express as a fraction in lowest terms, if it exists.

a. $\frac{3}{10} + \frac{2}{15}$ b. $\frac{8}{5} \div 5$ c. $\frac{99}{25} \times \frac{5}{3}$ d. $\frac{2}{5} \div \frac{0}{2}$

2(2 pts). Rationalize the denominator and simplify the result: $\frac{3}{2+\sqrt{7}}$

3(6 pts). Rewrite the expression in simplest radical form or state that it does not exist.

a. $\sqrt{108}$ b. $\sqrt[3]{-378}$ c. $24^{3/2}$

4(9 pts). Rewrite the expression without parentheses. Write your answer to part a. without negative exponents. Write your answer to part b. without fractions (except possibly in an exponent).

a. $2x^3(3y^2)^3x^{-10}3^{-1}$ b. $\frac{\left(\frac{r}{s}\right)\left(\frac{s^2}{r^3}\right)}{\frac{3}{s} - \frac{7}{s}}$ c. $(uv^{-2}u^{-3}v^8)^{3/2}$

5(10 pts). Factor the polynomial completely over the real numbers.

a. $8x^3 + 27y^3$ b. $3x^5 - 24x^2$ c. $3x^3 - 8x^2 - 12x + 32$

6(4 pts). Simplify the rational function: $\frac{12x^2 - 11x - 15}{3x^2 + 4x - 15}$

7(15 pts). Perform the operation indicated and simplify the resulting rational function.

a. $\frac{1}{x-4} - \frac{3}{x-3}$ b. $\frac{x+2}{x^2+4} \cdot \frac{x-2}{x^2+4x+4}$
c. $\frac{1}{2x^2-11x+15} + \frac{1}{2x^2-9x+10}$ d. $\frac{\frac{2x+3}{3x-4} + 5}{x-1}$

1a. (Source: BootCamp.3d, 5m, 6p, 6.27) $\frac{3}{10} + \frac{2}{15} = \frac{3}{2 \cdot 5} + \frac{2}{3 \cdot 5} = \frac{3}{2 \cdot 5} \cdot \frac{3}{3} + \frac{2}{3 \cdot 5} \cdot \frac{2}{2} = \frac{9+4}{2 \cdot 3 \cdot 5} = \frac{13}{30}.$

1b. $\frac{8}{5} \div 5 = \frac{8}{5} \cdot \frac{1}{5} = \frac{8}{25}.$

1c. $\frac{99}{25} \times \frac{5}{3} = \frac{3 \cdot 33 \cdot 5}{5 \cdot 5 \cdot 3} = \frac{33}{5}.$

1d. $\frac{0}{2} = 0$, so $\frac{2}{5} \div \frac{0}{2}$ does not exist.

2(2 pts). (Source: BootCamp.8b) Multiply top and bottom by the conjugate.

$$\frac{3}{2+\sqrt{7}} \cdot \frac{2-\sqrt{7}}{2-\sqrt{7}} = \frac{3(2-\sqrt{7})}{2^2-\sqrt{7}^2} = \frac{3(2-\sqrt{7})}{4-7} = \frac{3(2-\sqrt{7})}{-3} = -(2-\sqrt{7}), \text{ or } -2+\sqrt{7}.$$

3(6 pts). (Source: BootCamp.9c, 10m, 11d, 11y) It helps to factor the radicand (the number under the radical) into prime numbers. This might take a few steps, as in part b.

3a. $\sqrt{108} = \sqrt{2^2 3^3} = \sqrt{2^2 3^2 3} = 2 \cdot 3\sqrt{3} = 6\sqrt{3}.$

3b. $\sqrt[3]{-378} = \sqrt[3]{-3 \cdot 126} = \sqrt[3]{-3 \cdot 3 \cdot 42} = \sqrt[3]{-3^2 \cdot 6 \cdot 7} = \sqrt[3]{-3^3 \cdot 2 \cdot 7} = -3\sqrt[3]{14}.$

3c. $24^{3/2} = (2^3 \cdot 3)^{3/2} = 2^{9/2} \cdot 3^{3/2} = 2^4 \cdot 3^1 \cdot 2^{1/2} \cdot 3^{1/2} = 48 \cdot 6^{1/2}$, or $48\sqrt{6}.$

4a. (Source: BootCamp.12eir) $2x^3(3y^2)^3x^{-10}3^{-1} = 2x^33^3(y^2)^3x^{-10}3^{-1} = 2x^33^3y^6x^{-10}3^{-1} = 2x^{-7}3^2y^6.$ Without negative exponents, this is $\frac{18y^6}{x^7}.$

4b. $\left(\frac{r}{s} \cdot \frac{s^2}{r^3}\right) \div \frac{-4}{s} = \frac{s^1}{r^2} \div \frac{-4}{s} = \frac{s^1}{r^2} \cdot \frac{s}{-4} = \frac{s^2}{-4r^2}.$ With negative exponents, we can rewrite this as $-4^{-1}r^{-2}s^2.$

4c. $(uv^{-2}u^{-3}v^8)^{3/2} = (u^{1-3}v^{-2+8})^{3/2} = (u^{-2}v^6)^{3/2} = u^{-2 \cdot 3/2}v^{6 \cdot 3/2} = u^{-3}v^9$, or $\frac{v^9}{u^3}.$

5a. (Source: BootCamp.13km, 15g)

$$8x^3+27y^3 = (2x)^3+(3y)^3 = (2x+3y)((2x)^2-(2x)(3y)+(3y)^2) = (2x+3y)(4x^2-6xy+9y^2).$$

The quadratic can't be factored further.

5b. Common factor. $3x^5 - 24x^2 = 3x^2(x^3 - 8) = 3x^2(x^3 - 2^3) = 3x^2(x - 2)(x^2 + 2x + 4).$

The quadratic can't be factored further.

5c. Grouping. $3x^3 - 8x^2 - 12x + 32 = x^2(3x - 8) - 4(3x - 8) = (x^2 - 4)(3x - 8) = (x - 2)(x + 2)(3x - 8).$

6. (Source: BootCamp.18d) Factor the numerator and denominator, and cancel any common factors. $\frac{12x^2-11x-15}{3x^2+4x-15} = \frac{(3x-5)(4x+3)}{(3x-5)(x+3)} = \frac{4x+3}{x+3}.$

7a. (Source: BootCamp.19cj, 20cm) $\frac{(x-3)}{(x-3)} \frac{1}{(x-4)} - \frac{3}{(x-3)} \frac{(x-4)}{(x-4)} = \frac{(x-3)-3(x-4)}{(x-3)(x-4)} = \frac{-2x+9}{(x-3)(x-4)}.$ Avoid the common mistake of cancelling $(x-3)$ or $(x-4)$ at $\star$. Neither of these is a factor of the top.

7b. $\frac{x+2}{x^2+4} \cdot \frac{x-2}{x^2+4x+4} = \frac{x+2}{x^2+4} \cdot \frac{x-2}{(x+2)^2} = \frac{x-2}{(x^2+4)(x+2)}.$ Note that $x^2 + 4 \neq (x + 2)^2.$

7c. $\frac{1}{2x^2-11x+15} + \frac{1}{2x^2-9x+10} = \frac{1}{(x-3)(2x-5)} + \frac{1}{(2x-5)(x-2)}$
 $= \frac{(x-2)}{(x-3)(2x-5)(x-2)} + \frac{(x-3)}{(x-3)(2x-5)(x-2)} = \frac{2x-5}{(x-3)(2x-5)(x-2)} = \frac{1}{(x-3)(x-2)}.$

7d. $\frac{\frac{2x+3}{3x-4}+5}{x-1} = \left(\frac{2x+3}{3x-4} + \frac{5(3x-4)}{3x-4}\right) \div (x-1) = \frac{2x+3+5(3x-4)}{3x-4} \cdot \frac{1}{(x-1)} = \frac{17x-17}{(3x-4)(x-1)} = \frac{17(x-1)}{(3x-4)(x-1)} = \frac{17}{3x-4}.$