MATH 101–01 (Kunkle), Exam 2 — Name: ____________________

200 pts, 3.5 hours — June 2, 2014 —

No notes, books, electronic devices, or outside materials of any kind.

Read each problem carefully and simplify your answers. Guessing the correct answer is not sufficient for full credit. Supporting work will be required on every problem unless otherwise indicated.

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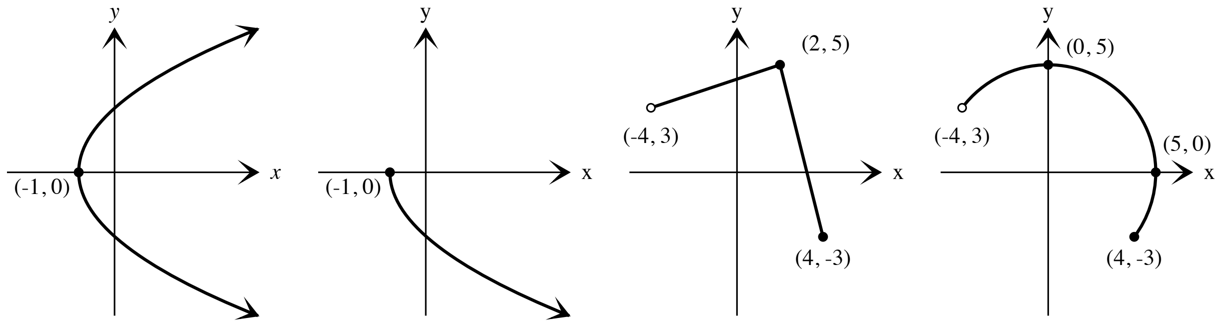
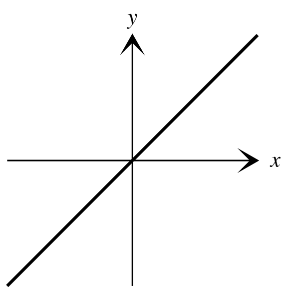
1 (12 pts). Indicate whether each of the graphs below is the graph of a function. For those that are, find the domain and the range. Write your answers in interval form.



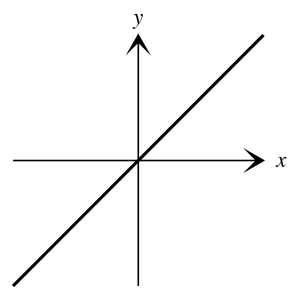
☐ Not a function	☐ Not a function	☐ Not a function	☐ Not a function
☐ A function domain = ________ range = ________	☐ A function domain = ________ range = ________	☐ A function domain = ________ range = ________	☐ A function domain = ________ range = ________

2 (8 pts). Identify the formula for the function from its graph.

For instance, if you were given this graph, this would be the correct response.

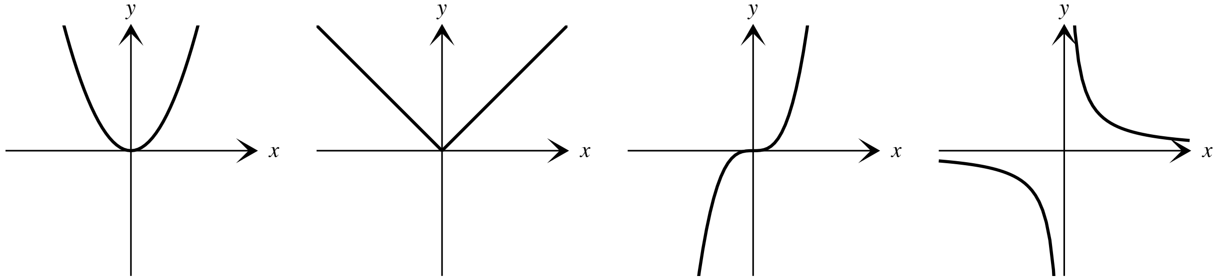


$p(x) =$



$p(x) = x$

There might be more than one correct answer, but piecewise-defined functions are not acceptable.



$f(x) =$ $\quad g(x) =$ $\quad h(x) =$ $\quad k(x) =$

3 (18 pts). Let $g(x) = -(x+2)^2 + 5$. Find the following.

a. The vertex of the graph of $g(x)$.

b. The equation of the axis of symmetry of the graph of $g(x)$.

c. All intercepts of the graph of $g(x)$.

d. The maximum or minimum values of $g(x)$ or state that they do not exist.

e. A sketch of the graph of $g(x)$, reflecting your answers to parts a.-d.

4 (8 pts). A new piece of office machinery is worth \$5600.00 today but will steadily loose value until it's worth \$0 in 8 years. Let V stand for the value of the machine in dollars and t stand for its age in years. Today, for instance, $V = \$5600.00$ and $t = 0$. Express V as a linear function of t.

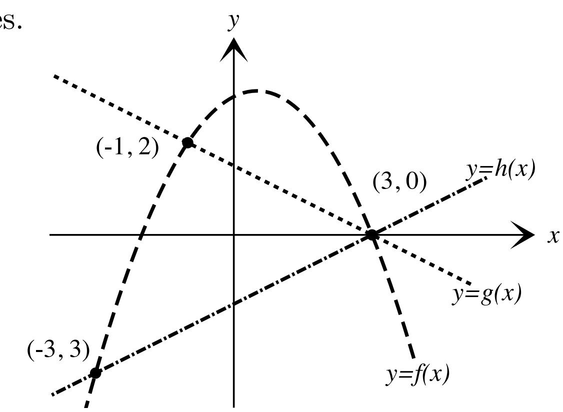
5 (8 pts). Sketch the graph of the function $f(x) = \begin{cases} -1 & \text{if } x \leq -2 \\ 1 + \frac{1}{2}x & \text{if } -2 < x < 0 \\ \sqrt{x} & \text{if } 0 \leq x \end{cases}$

Include the coordinates of all intercepts and of any $\circ$ or $\bullet$ points in the graph.

6 (13 pts). Find the domains of the following functions.

a. $\frac{2}{3x^2+8x-9}$ b. $\frac{3x^2+8x-9}{x^2+9}$ c. $\log_2(x-7)$

7 (6 pts). Use the figure to solve for x in the inequalities. Write the solution set in interval form.



a. $f(x) > h(x)$

b. $f(x) > g(x) > h(x)$

8 (10 pts). Let $f(x) = \frac{5x^2+1}{2x-3}$. Find the following.

a. $f(-2)$ b. $f(-x)$ c. $f(x+1)$

9 (8 pts). Evaluate the logs.

a. $\log_5 125 =$ b. $\log_3 \frac{1}{9} =$ c. $\log_3 \sqrt[5]{81} =$ d. $\log_{10} 1 =$

10 (18 pts). Solve the inequalities. Write your solution sets in interval form.

a. $2x^2 - 7x + 4 > 0$ b. $\frac{x-3}{x+2} \leq -2$ c. $x^2(x-1)(x+3)^3 > 0$

11 (6 pts). Sketch the graph of $y = x^2(x-1)(x+3)^3$. Your drawing need not be to scale, but it should clearly show how and where the graph intersects the x-axis.

12 (4 pts). Use the table below to find the following.

x	-3	-2	-1	0	1	2
$f(x)$	2	-1	1	-2	0	-3
$g(x)$	1	2	0	-1	-3	2

a. $(f \circ g)(-2) =$ b. $(f \circ f)(-1) =$

13 (6 pts). Determine whether the functions f and g from Problem 12 are one-to-one. If either is one-to-one, evaluate its inverse function at -3.

☐ f is not one-to-one	☐ g is not one-to-one
☐ f is one-to-one and $f^{-1}(-3) =$ __________	☐ g is one-to-one and $g^{-1}(-3) =$ __________

14 (18 pts). Let $f(x) = 3x + 2$ and $g(x) = \sqrt{x-1}$. Find the following functions and the domain of each.

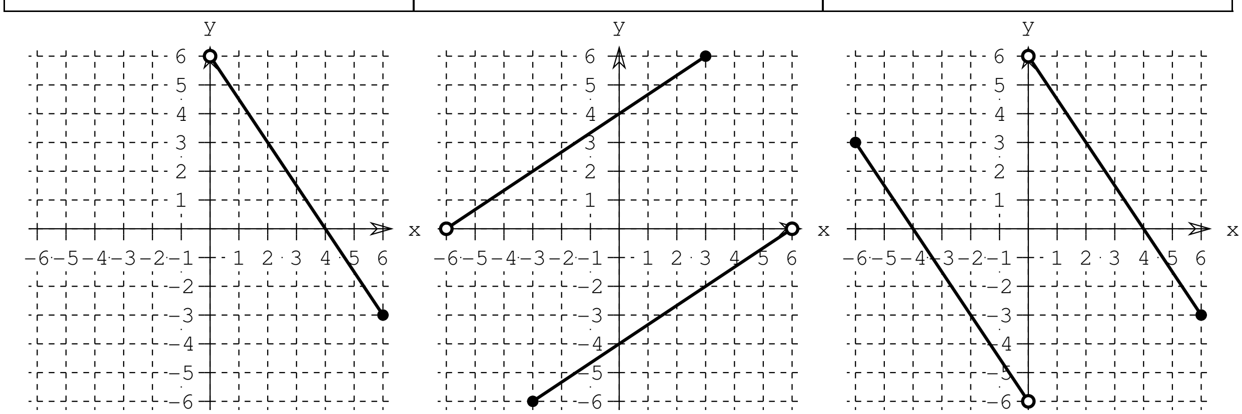
a. $(f \cdot g)(x)$ b. $\left(\frac{f}{g}\right)(x)$ c. $(g \circ f)(x)$

15 (12 pts). Solve each equation for x.

a. $8^{1-2x} = 4^{x+3}$ b. $\log_3(x+1) = 2$ c. $e^x = 10$.

16 (3 pts). Pick the best description for each graph.

☐ Not a function ☐ A function, but not one-to-one ☐ A one-to-one function	☐ Not a function ☐ A function, but not one-to-one ☐ A one-to-one function	☐ Not a function ☐ A function, but not one-to-one ☐ A one-to-one function

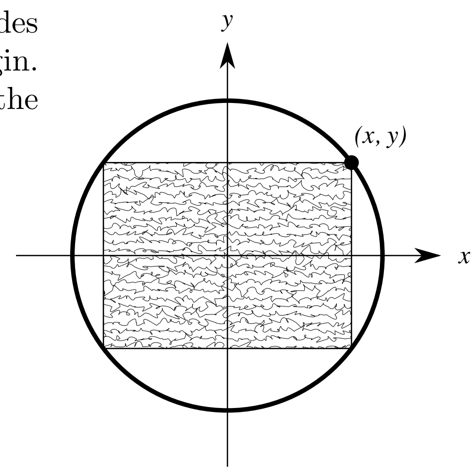


17 (5 pts). For each one-to-one function $f(x)$ that you found in Problem 16 above, graph $y = f^{-1}(x)$ on the same axes.

18 (6 pts). Find $f^{-1}(x)$ if $f(x) = x^3 - 4$.

19 (4 pts). Let $H(x) = \sqrt{x^2+1}$. Find functions $f(x)$ and $g(x)$ so that $H(x) = (f \circ g)(x)$. There are many correct answers, but you may not use either $f(x) = x$ or $g(x) = x$.

20 (8 pts). A rectangle with horizontal and vertical sides is inscribed in the circle of radius 3 centered at the origin. Let (x, y) be the vertex of the rectangle that meets the circle in quadrant I.
Express the area A of the rectangle as a function of x.



21 (8 pts). Rewrite $f(x) = -3x^2 + 18x + 15$ in the form $f(x) = a(x - h)^2 + k$.

22 (3 pts). Suppose $x = \ln 3$ and $y = \ln 2$. Express the following in terms of x and y.

a. $\ln 6$ b. $\ln 32$

23 (8 pts). Simplify the expression, or state that it does not exist.

a. $e^{\ln 7}$ b. $\log_2 2^{301}$ c. $e^{\ln(-3)}$ d. $\log_3 3^{(-9.5)}$

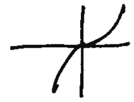
Solns 6-2-2014

1.  fails VLT. Not a function.

(-1,0)  function. domain = x's $= (-1, \infty)$. Range = y's $= (-\infty, 0)$.

 function. domain $= (-4, 4]$. Range $= [-3, 5]$.

 fails VLT. Not a function.

2.  $f(x) = x^2$  $g(x) = |x|$  $h(x) = x^3$  $k(x) = \frac{1}{x}$

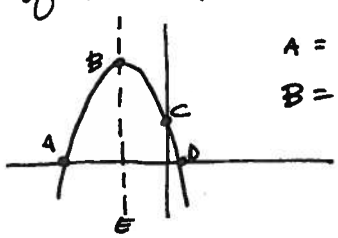
3. a. vertex @ $(-2, 5)$. b. axis is $x = -2$.

c. $x = 0 \Rightarrow y = -(0+2)^2 + 5 = -4+5 = 1$. y-intercept is $(0,1)$

$y = 0 = -(x+2)^2 + 5 \Rightarrow (x+2)^2 = 5$; $x+2 = \pm\sqrt{5}$; $x = -2 \pm \sqrt{5}$

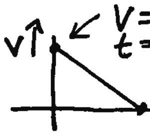
x-intercepts are $(-2-\sqrt{5}, 0)$ and $(-2+\sqrt{5}, 0)$.

d. graph opens down: so maximum value = y-coordinate of vertex = 1. g(x) has no minimum value

e. 

$A = (-2-\sqrt{5}, 0)$ $C = (0, 1)$ $E: x = -2$

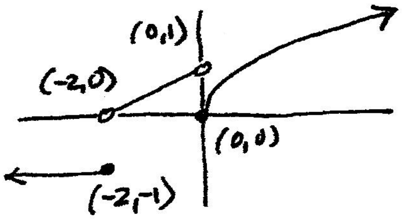
$B = (-2, 5)$ $D = (-2+\sqrt{5}, 0)$

4. Graph of V vs t: 

$V = 5600$, $t = 0$

$V = 0$, $t = 8$. slope $= \frac{\Delta V}{\Delta t} = \frac{-5600}{8} = -700$

slope-intercept form: $V = 5600 - 700t$.

5. 

6a. Denominator $=0$ when $3x^2+8x-9=0$, or

$$x = \frac{-8 \pm \sqrt{64-4\cdot 3(-9)}}{6} = \frac{-8\pm\sqrt{172}}{6}$$

$$= \frac{-8 \pm \sqrt{4}\sqrt{43}}{6} = \frac{2(-4\pm\sqrt{43})}{2\cdot 3}.$$

Domain: all real # except $(-4\pm\sqrt{43})/3$.

6b. Denominator x^2+9 has no real zeros (b.c. $x^2=-9$ has no real solutions). Domain = all real numbers.

6c. Can only take ln of positive numbers: need $x-7>0$, or $x>7$.

7a. $f>h$: $-3<x<3$, or $(-3,3)$

b. $f>g>h$: $-1<x<3$, or $(-1,3)$.

8a. $$f(-2) = \frac{5(-2)^2+1}{2(-2)-3} = \frac{5\cdot 4+1}{-4-3} = \frac{21}{-7} = -3.$$

8b. $$f(-x) = \frac{5(-x)^2+1}{2(-x)-3} = \frac{5(-1)^2(x)^2+1}{-2x-3} = \frac{5x^2+1}{-2x-3}$$

8c. $$f(x+1) = \frac{5(x+1)^2+1}{2(x+1)-3} = \frac{5(x^2+2x+1)+1}{2x+2-3} = \frac{5x^2+10x+6}{2x-1}$$

(Note $5x^2+1$ and $5x^2+10x+6$ do not factor.)

9a. $125=5^3$, so $\log_5 125 = 3$.

b. $\frac{1}{9} = 3^{-2}$, so $\log_3 \frac{1}{9} = -2$

c. $\sqrt[5]{81} = 3^{4/5}$, so $\log_3 \sqrt[5]{81} = 4/5$.

d. $1 = 10^0$, so $\log_{10} 1 = 0$.

10 a. Find zeros : $2x^2-7x+4=0$ @ $x=\dfrac{7\pm\sqrt{49-4\cdot 2\cdot 4}}{4}=\dfrac{7\pm\sqrt{17}}{4}$

parabola opens up :

$\dfrac{7-\sqrt{17}}{4}$ $\dfrac{7+\sqrt{17}}{4}$

$y=2x^2-7x+4$ must be positive here :

$\left(-\infty, \dfrac{7-\sqrt{17}}{4}\right)\cup\left(\dfrac{7+\sqrt{17}}{4}, \infty\right)$

(or test pts $x=-10$ $\quad 7/4$ $\quad 10$
$y=274$ $\quad -17/8$ $\quad 134$)

b. Get zero on one side, factor the other, make a sign chart.
Do not multiply by $x+2$, since $x+2$ is sometimes $+$, sometimes $-$.

$\dfrac{x-3}{x+2}+2\le 0$; $\quad \dfrac{x-3}{x+2}+\dfrac{2}{1}\dfrac{(x+2)}{(x+2)}=\dfrac{x-3+2(x+2)}{x+2}=\dfrac{3x+1}{x+2}$.

$3x+1$	$- - -$	$- - -$	0	$+ + +$
$x+2$	$- - -$	0 $+ + +$		$+ + +$
$(3x+1)/(x+2)$	$+ +$	* $- - -$	0	$+ +$
x		-2	$-1/3$	

* Does not exist

$\dfrac{3x+1}{x+2}\le 0$ @ $- - -$ or 0 in chart.

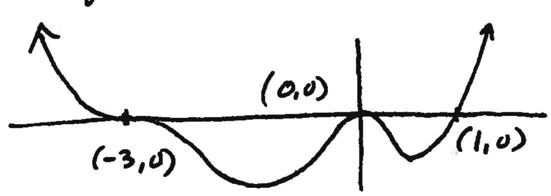
Sol'n set : $(-2, -1/3]$.

c.

x^2	$+ + + +$		$+ +$	0	$+ + + + +$		← x^2 always ≥ 0
$x-1$	$- - - -$		$- - -$		$- - - -$	0 $+ + +$	
$(x+3)^3$	$- - - -$	0	$+ + + +$		$+ + + + +$		← sign of $(x+3)^3$ is same as sign of $(x+3)$.
$x^2(x-1)(x+3)^3$	$+ +$	0	$- - -$	0	$- - -$	0 $+ + +$	
		-3		0		1	

>0 : $++$'s only : $(-\infty,-3)\cup(1,\infty)$

11. See sign chart in 10c.



Graph crosses x-axis where y changes sign : @ $x=1,-3$.
Graph touches x-axis @ $x=0$ but does not cross there.

12. $(f\circ g)(-2)=f(g(-2))=f(2)=-3$
$(f\circ f)(-1)=f(f(-1))=f(1)=0$

13. f is one-to-one, because no two x's have the same y.
$f^{-1}(-3)=2$ because $f(2)=-3$.
g is not one-to-one because $g(-2)=g(2)$ $(=2)$.

14a. $(f \cdot g)(x) = f(x) \cdot g(x) = (3x+2)\sqrt{x-1}$.
Domain. need $x-1 \geq 0$; $x \geq 1$ (i.e. $[1,\infty)$)

14b. $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{3x+2}{\sqrt{x-1}}$, As in a., need $x \geq 1$, but also need $\sqrt{x-1} \neq 0$, so $x \neq 1$. Domain $= (1,\infty)$.

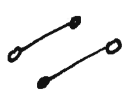
14c. $(g \circ f)(x) = g(f(x)) = g(3x+2) = \sqrt{(3x+2)-1} = \sqrt{3x+1}$.
Domain: need $3x+1 \geq 0$; $3x \geq -1$; $x \geq -1/3$. ($[-1/3,\infty)$)

15a. $8^{1-2x} = 4^{x+3}$; $(2^3)^{1-2x} = (2^2)^{x+3}$; $2^{3(1-2x)} = 2^{2(x+3)}$
$\Rightarrow 3(1-2x) = 2(x+3)$; $3-6x = 2x+6$; $-3 = 8x$;
$x = -3/8$.

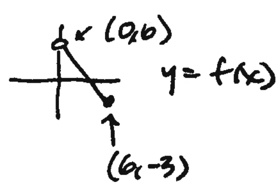
15b. $\log_3(x+1) = 2 \Rightarrow x+1 = 3^2 = 9$; $x = 8$.

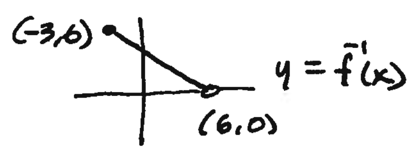
15c. $e^x = 10 \Rightarrow x = \ln 10$.

16.  passes both VLT and HLT; a one-to-one function.

 fails VLT. Not a function.

 passes VLT but fails HLT. A function, but not one-to-one

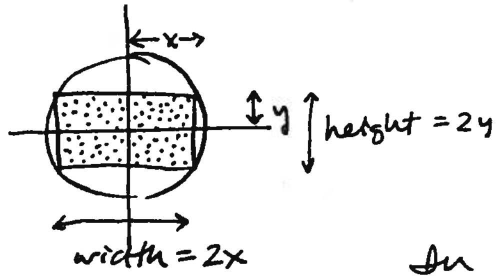
17. 



18. $y = x^3 - 4$ (solve for x) $y+4 = x^3$; $x = \sqrt[3]{y+4} = f^{-1}(y)$.
$f^{-1}(x) = \sqrt[3]{x+4}$.

19. Sol'n one: $f(x) = \sqrt{x}$ & $g(x) = x^2+1$. Then $f(g(x)) = \sqrt{x^2+1}$.
Sol'n two: $f(x) = \sqrt{x+1}$ $g(x) = x^2$. " " " .

20.



circle : $x^2+y^2=3^2$. Solve for y:

$y^2=9-x^2$; $y=\pm\sqrt{9-x^2}$.

In quadrant I, $y>0$, so $y=+\sqrt{9-x^2}$.

$A = \text{width}\cdot\text{height} = 2x\cdot 2y = 4xy = 4x\sqrt{9-x^2}$.

21. $f(x) = -3x^2+18x+15$. Divide by -3: $-\frac{1}{3}f(x) = x^2-6x-5$

Complete the square: $-\frac{1}{3}f(x) = x^2-6x+9-9-5 = (x-3)^2-14$.

Mult. by -3: $f(x) = -3(x-3)^2-14(-3) = -3(x-3)^2+42$.

22. a. $\ln 6 = \ln 3\cdot 2 = \ln 3 + \ln 2 = x+y$

b. $\ln 32 = \ln 2^5 = 5\ln 2 = 5y$.

23. a. $e^{\ln 7} = 7$ b. $\log_2 2^{301} = 301$

c. $\ln x$ undefined if $x \le 0$. $\ln(-3)$ DNE, so $e^{\ln(-3)}$ DNE.

d. $\log_3(3^{-9.5}) = -9.5$.